

ALERT FOR GOVERNMENT JOBS

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1. NUMBERS

IMPORTANT FACTS AND FORMULAE

I. **Numeral** : In Hindu Arabic system, we use ten symbols 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 called **digits** to represent any number.

A group of digits, denoting a number is called a **numeral**.

We represent a number, say 689745132 as shown below :

Ten Crores (10^8)	Crores (10^7)	Ten Lacs (Millions) (10^6)	Lacs (10^5)	Ten Thousands (10^4)	Thousands (10^3)	Hundreds (10^2)	Tens (10^1)	Units (10^0)
6	8	9	7	4	5	1	3	2

We read it as : 'Sixty-eight crores, ninety-seven lacs, forty-five thousand, one hundred and thirty-two'.

II. **Place Value or Local Value of a Digit in a Numeral** :

In the above numeral :

Place value of 2 is $(2 \times 1) = 2$; Place value of 3 is $(3 \times 10) = 30$;

Place value of 1 is $(1 \times 100) = 100$ and so on.

Place value of 6 is $6 \times 10^8 = 600000000$.

III. **Face Value** : The **face value** of a digit in a numeral is the value of the digit itself at whatever place it may be. In the above numeral, the face value of 2 is 2; the face value of 3 is 3 and so on.

IV. **TYPES OF NUMBERS**

1. **Natural Numbers** : Counting numbers 1, 2, 3, 4, 5, are called **natural numbers**.

2. **Whole Numbers** : All counting numbers together with zero form the set of **whole numbers**. Thus,

(i) 0 is the only whole number which is not a natural number.

(ii) Every natural number is a whole number.

3. **Integers** : All natural numbers, 0 and negatives of counting numbers i.e., {....., -3, -2, -1, 0, 1, 2, 3,} together form the set of integers.

(i) **Positive Integers** : {1, 2, 3, 4,} is the set of all positive integers.

(ii) **Negative Integers** : {-1, -2, -3,} is the set of all negative integers.

(iii) **Non-Positive and Non-Negative Integers** : 0 is neither positive nor negative. So, {0, 1, 2, 3,} represents the set of non-negative integers, while {0, -1, -2, -3,} represents the set of non-positive integers.

4. **Even Numbers** : A number divisible by 2 is called an even number. e.g., 2, 4, 6, 8, 10, etc.

5. **Odd Numbers** : A number not divisible by 2 is called an odd number. e.g., 1, 3, 5, 7, 9, 11, etc.

6. **Prime Numbers** : A number greater than 1 is called a prime number, if it has exactly two factors, namely 1 and the number itself.

Prime numbers upto 100 are : 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97.

Prime numbers Greater than 100 : Let p be a given number greater than 100. To find out whether it is prime or not, we use the following method :

Find a whole number nearly greater than the square root of p . Let $k > \sqrt{p}$. Test whether p is divisible by any prime number less than k . If yes, then p is not prime. Otherwise, p is prime.

e.g., We have to find whether 191 is a prime number or not. Now, $14 > \sqrt{191}$.

Prime numbers less than 14 are 2, 3, 5, 7, 11, 13.

191 is not divisible by any of them. So, 191 is a prime number.

7. **Composite Numbers** : Numbers greater than 1 which are not prime, are known as composite numbers. e.g., 4, 6, 8, 9, 10, 12.

Note : (i) 1 is neither prime nor composite.

(ii) 2 is the only even number which is prime.

(iii) There are 25 prime numbers between 1 and 100.

8. **Co-primes** : Two numbers a and b are said to be co-primes, if their H.C.F. is 1. e.g., (2, 3), (4, 5), (7, 9), (8, 11), etc. are co-primes.

V. TESTS OF DIVISIBILITY

1. **Divisibility By 2** : A number is divisible by 2, if its unit's digit is any of 0, 2, 4, 6, 8.

Ex. 84932 is divisible by 2, while 65935 is not.

2. **Divisibility By 3** : A number is divisible by 3, if the sum of its digits is divisible by 3.

Ex. 592482 is divisible by 3, since sum of its digits = $(5 + 9 + 2 + 4 + 8 + 2) = 30$, which is divisible by 3.

But, 864329 is not divisible by 3, since sum of its digits = $(8 + 6 + 4 + 3 + 2 + 9) = 32$, which is not divisible by 3.

3. **Divisibility By 4** : A number is divisible by 4, if the number formed by the last two digits is divisible by 4.

Ex. 892648 is divisible by 4, since the number formed by the last two digits is 48, which is divisible by 4.

But, 749282 is not divisible by 4, since the number formed by the last two digits is 82, which is not divisible by 4.

4. **Divisibility By 5** : A number is divisible by 5, if its unit's digit is either 0 or 5. Thus, 20820 and 50345 are divisible by 5, while 30934 and 40946 are not.

5. **Divisibility By 6** : A number is divisible by 6, if it is divisible by both 2 and 3.

Ex. The number 35256 is clearly divisible by 2.

Sum of its digits = $(3 + 5 + 2 + 5 + 6) = 21$, which is divisible by 3.

Thus, 35256 is divisible by 2 as well as 3. Hence, 35256 is divisible by 6.

6. **Divisibility By 8 :** A number is divisible by 8, if the number formed by the last three digits of the given number is divisible by 8.

Ex. 953360 is divisible by 8, since the number formed by last three digits is 360, which is divisible by 8.

But, 529418 is not divisible by 8, since the number formed by last three digits is 418, which is not divisible by 8.

7. **Divisibility By 9 :** A number is divisible by 9, if the sum of its digits is divisible by 9.

Ex. 60732 is divisible by 9, since sum of digits = $(6 + 0 + 7 + 3 + 2) = 18$, which is divisible by 9.

But, 68956 is not divisible by 9, since sum of digits = $(6 + 8 + 9 + 5 + 6) = 34$, which is not divisible by 9.

8. **Divisibility By 10 :** A number is divisible by 10, if it ends with 0.

Ex. 96410, 10480 are divisible by 10, while 96375 is not.

9. **Divisibility By 11 :** A number is divisible by 11, if the difference of the sum of its digits at odd places and the sum of its digits at even places, is either 0 or a number divisible by 11.

Ex. The number 4832718 is divisible by 11, since :

(sum of digits at odd places) – (sum of digits at even places)

= $(8 + 7 + 3 + 4) - (1 + 2 + 8) = 11$, which is divisible by 11.

10. **Divisibility By 12 :** A number is divisible by 12, if it is divisible by both 4 and 3.

Ex. Consider the number 34632.

(i) The number formed by last two digits is 32, which is divisible by 4.

(ii) Sum of digits = $(3 + 4 + 6 + 3 + 2) = 18$, which is divisible by 3.

Thus, 34632 is divisible by 4 as well as 3. Hence, 34632 is divisible by 12.

11. **Divisibility By 14 :** A number is divisible by 14, if it is divisible by 2 as well as 7.

12. **Divisibility By 15 :** A number is divisible by 15, if it is divisible by both 3 and 5.

13. **Divisibility By 16 :** A number is divisible by 16, if the number formed by the last 4 digits is divisible by 16.

Ex. 7957536 is divisible by 16, since the number formed by the last four digits is 7536, which is divisible by 16.

14. **Divisibility By 24 :** A given number is divisible by 24, if it is divisible by both 3 and 8.

15. **Divisibility By 40 :** A given number is divisible by 40, if it is divisible by both 5 and 8.

16. **Divisibility By 80 :** A given number is divisible by 80, if it is divisible by both 5 and 16.

Note : If a number is divisible by p as well as q , where p and q are co-primes, then the given number is divisible by pq .

If p and q are not co-primes, then the given number need not be divisible by pq , even when it is divisible by both p and q .

Ex. 36 is divisible by both 4 and 6, but it is not divisible by $(4 \times 6) = 24$, since 4 and 6 are not co-primes.

VI. MULTIPLICATION BY SHORT CUT METHODS

1. Multiplication By Distributive Law :

$$(i) a \times (b + c) = a \times b + a \times c \quad (ii) a \times (b - c) = a \times b - a \times c.$$

$$\text{Ex. (i) } 567958 \times 99999 = 567958 \times (100000 - 1)$$

$$= 567958 \times 100000 - 567958 \times 1$$

$$= (56795800000 - 567958) = 56795232042.$$

$$(ii) 978 \times 184 + 978 \times 816 = 978 \times (184 + 816) = 978 \times 1000 = 978000.$$

2. Multiplication of a Number By 5^n : Put n zeros to the right of the multiplicand and divide the number so formed by 2^n .

$$\text{Ex. } 975436 \times 625 = 975436 \times 5^4 = \frac{9754360000}{16} = 609647500.$$

VII. BASIC FORMULAE

$$1. (a + b)^2 = a^2 + b^2 + 2ab$$

$$2. (a - b)^2 = a^2 + b^2 - 2ab$$

$$3. (a + b)^2 - (a - b)^2 = 4ab$$

$$4. (a + b)^2 + (a - b)^2 = 2(a^2 + b^2)$$

$$5. (a^2 - b^2) = (a + b)(a - b)$$

$$6. (a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$$

$$7. (a^3 + b^3) = (a + b)(a^2 - ab + b^2) \quad 8. (a^3 - b^3) = (a - b)(a^2 + ab + b^2)$$

$$9. (a^3 + b^3 + c^3 - 3abc) = (a + b + c)(a^2 + b^2 + c^2 - ab - bc - ca)$$

$$10. \text{ If } a + b + c = 0, \text{ then } a^3 + b^3 + c^3 = 3abc.$$

VIII. DIVISION ALGORITHM OR EUCLIDEAN ALGORITHM

If we divide a given number by another number, then :

$$\text{Dividend} = (\text{Divisor} \times \text{Quotient}) + \text{Remainder}$$

IX. (i) $(x^n - a^n)$ is divisible by $(x - a)$ for all values of n .

(ii) $(x^n - a^n)$ is divisible by $(x + a)$ for all even values of n .

(iii) $(x^n + a^n)$ is divisible by $(x + a)$ for all odd values of n .

X. PROGRESSION

A succession of numbers formed and arranged in a definite order according to certain definite rule, is called a *progression*.

1. **Arithmetic Progression (A.P.)** : If each term of a progression differs from its preceding term by a constant, then such a progression is called an arithmetical progression. This constant difference is called the *common difference of the A.P.*

An A.P. with first term a and common difference d is given by $a, (a + d), (a + 2d), (a + 3d), \dots$

The n th term of this A.P. is given by $T_n = a + (n - 1)d$.

The sum of n terms of this A.P.

$$S_n = \frac{n}{2} [2a + (n - 1)d] = \frac{n}{2} (\text{first term} + \text{last term}).$$

SOME IMPORTANT RESULTS :

$$(i) (1 + 2 + 3 + \dots + n) = \frac{n(n + 1)}{2}$$

$$(ii) (1^2 + 2^2 + 3^2 + \dots + n^2) = \frac{n(n + 1)(2n + 1)}{6}$$

$$(iii) (1^3 + 2^3 + 3^3 + \dots + n^3) = \frac{n^2(n + 1)^2}{4}$$

2. Geometrical Progression (G.P.) : A progression of numbers in which every term bears a constant ratio with its preceding term, is called a *geometrical progression*. The constant ratio is called the common ratio of the G.P.

A G.P. with first term a and common ratio r is :

$$a, ar, ar^2, ar^3, \dots$$

In this G.P. $T_n = ar^{n-1}$.

$$\text{Sum of the } n \text{ terms, } S_n = \frac{a(1-r^n)}{(1-r)}.$$

SOLVED EXAMPLES

Ex. 1. Simplify : (i) $8888 + 888 + 88 + 8$

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(ii) $11992 - 7823 - 456$

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Sol. (i)

$$\begin{array}{r} 8888 \\ 888 \\ 88 \\ + 8 \\ \hline 9872 \end{array}$$

(ii) $11992 - 7823 - 456 = 11992 - (7823 + 456)$
 $= 11992 - 8279 = 3713.$

$$\begin{array}{r} 7823 \\ + 456 \\ \hline 8279 \end{array} \qquad \begin{array}{r} 11992 \\ - 8279 \\ \hline 3713 \end{array}$$

Ex. 2. What value will replace the question mark in each of the following equations

(i) $? - 1936248 = 1635773$

(ii) $8597 - ? = 7429 - 4358$

(Bank P.O. 2000

Sol. (i) Let $x - 1936248 = 1635773$. Then, $x = 1635773 + 1936248 = 3572021$.

(ii) Let $8597 - x = 7429 - 4358$.

Then, $x = (8597 + 4358) - 7429 = 12955 - 7429 = 5526$.

Ex. 3. What could be the maximum value of Q in the following equation ?

$5P9 + 3R7 + 2Q8 = 1114$

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Sol. We may analyse the given equation as shown :

Clearly, $2 + P + R + Q = 11$.

So, the maximum value of Q can be

$(11 - 2)$ i.e., 9 (when $P = 0, R = 0$).

1	2	
5	P	9
3	R	7
2	Q	8
11	1	4

Ex. 4. Simplify : (i) 5793405×9999 (ii) 839478×625

Sol. (i) $5793405 \times 9999 = 5793405 (10000 - 1) = 57934050000 - 5793405 = 57928256595$

(ii) $839478 \times 625 = 839478 \times 5^4 = \frac{8394780000}{16} = 524673750.$

Ex. 5. Evaluate : (i) $986 \times 137 + 986 \times 863$ (ii) $983 \times 207 - 983 \times 107$

Sol. (i) $986 \times 137 + 986 \times 863 = 986 \times (137 + 863) = 986 \times 1000 = 986000.$

(ii) $983 \times 207 - 983 \times 107 = 983 \times (207 - 107) = 983 \times 100 = 98300.$

Ex. 6. Simplify : (i) 1605×1605 (ii) 1398×1398

Sol. (i) $1605 \times 1605 = (1605)^2 = (1600 + 5)^2 = (1600)^2 + (5)^2 + 2 \times 1600 \times 5$
 $= 2560000 + 25 + 16000 = 2576025.$

(ii) $1398 \times 1398 = (1398)^2 = (1400 - 2)^2 = (1400)^2 + (2)^2 - 2 \times 1400 \times 2$
 $= 1960000 + 4 - 5600 = 1954404.$

Ex. 7. Evaluate : $(313 \times 313 + 287 \times 287).$

Sol. $(a^2 + b^2) = \frac{1}{2} [(a + b)^2 + (a - b)^2]$

$$\begin{aligned} \therefore (313)^2 + (287)^2 &= \frac{1}{2} [(313 + 287)^2 + (313 - 287)^2] = \frac{1}{2} [(600)^2 + (26)^2] \\ &= \frac{1}{2} (360000 + 676) = 180338. \end{aligned}$$

Ex. 8. Which of the following are prime numbers ?

(i) 241

(ii) 337

(iii) 391

(iv) 571

Sol. (i) Clearly, $16 > \sqrt{241}$. Prime numbers less than 16 are 2, 3, 5, 7, 11, 13.

241 is not divisible by any one of them.

\therefore 241 is a prime number.

(ii) Clearly, $19 > \sqrt{337}$. Prime numbers less than 19 are 2, 3, 5, 7, 11, 13, 17. 337 is not divisible by any one of them.

\therefore 337 is a prime number.

(iii) Clearly, $20 > \sqrt{391}$. Prime numbers less than 20 are 2, 3, 5, 7, 11, 13, 17, 19. We find that 391 is divisible by 17.

\therefore 391 is not prime.

(iv) Clearly, $24 > \sqrt{571}$. Prime numbers less than 24 are 2, 3, 5, 7, 11, 13, 17, 19, 23. 571 is not divisible by any one of them.

\therefore 571 is a prime number.

Ex. 9. Find the unit's digit in the product $(2467)^{153} \times (341)^{72}$.

Sol. Clearly, unit's digit in the given product = unit's digit in $7^{153} \times 1^{72}$.

Now, 7^4 gives unit digit 1.

$\therefore 7^{152}$ gives unit digit 1.

$\therefore 7^{153}$ gives unit digit $(1 \times 7) = 7$. Also, 1^{72} gives unit digit 1.

Hence, unit's digit in the product = $(7 \times 1) = 7$.

Ex. 10. Find the unit's digit in $(264)^{102} + (264)^{103}$.

(S.S.C. 1999)

Sol. Required unit's digit = unit's digit in $(4)^{102} + (4)^{103}$.

Now, 4^2 gives unit digit 6.

$\therefore (4)^{102}$ gives unit digit 6.

$(4)^{103}$ gives unit digit of the product (6×4) i.e., 4.

Hence, unit's digit in $(264)^{102} + (264)^{103}$ = unit's digit in $(6 + 4) = 0$.

Ex. 11. Find the total number of prime factors in the expression $(4)^{11} \times (7)^5 \times (11)^2$.

Sol. $(4)^{11} \times (7)^5 \times (11)^2 = (2 \times 2)^{11} \times (7)^5 \times (11)^2 = 2^{22} \times 7^5 \times 11^2$.

\therefore Total number of prime factors = $(22 + 5 + 2) = 29$.

Ex. 12. Simplify : (i) $896 \times 896 - 204 \times 204$

(ii) $387 \times 387 + 114 \times 114 + 2 \times 387 \times 114$

(iii) $81 \times 81 + 68 \times 68 - 2 \times 81 \times 68$

Sol. (i) Given exp. = $(896)^2 - (204)^2 = (896 + 204)(896 - 204) = 1100 \times 692 = 761200$.

(ii) Given exp. = $(387)^2 + (114)^2 + 2 \times 387 \times 114$

= $a^2 + b^2 + 2ab$, where $a = 387$, $b = 114$

= $(a + b)^2 = (387 + 114)^2 = (501)^2 = 251001$.

(iii) Given exp. = $(81)^2 + (68)^2 - 2 \times 81 \times 68 = a^2 + b^2 - 2ab$, where $a = 81$, $b = 68$

= $(a - b)^2 = (81 - 68)^2 = (13)^2 = 169$.

Ex. 13. Which of the following numbers is divisible by 3 ?

(i) 541326

(ii) 5967013

Sol. (i) Sum of digits in 541326 = $(5 + 4 + 1 + 3 + 2 + 6) = 21$, which is divisible by 3. Hence, 541326 is divisible by 3.

(ii) Sum of digits in 5967013 = $(5 + 9 + 6 + 7 + 0 + 1 + 3) = 31$, which is not divisible by 3.

Hence, 5967013 is not divisible by 3.

Ex. 14. What least value must be assigned to * so that the number $197*5462$ is divisible by 9 ?

Sol. Let the missing digit be x .

Sum of digits = $(1 + 9 + 7 + x + 5 + 4 + 6 + 2) = (34 + x)$.

For $(34 + x)$ to be divisible by 9, x must be replaced by 2.

Hence, the digit in place of * must be 2.

Ex. 15. Which of the following numbers is divisible by 4 ?

(i) 67920594

(ii) 618703572

Sol. (i) The number formed by the last two digits in the given number is 94, which is not divisible by 4.

Hence, 67920594 is not divisible by 4.

(ii) The number formed by the last two digits in the given number is 72, which is divisible by 4.

Hence, 618703572 is divisible by 4.

Ex. 16. Which digits should come in place of * and \$ if the number 62684*\$ is divisible by both 8 and 5 ?

Sol. Since the given number is divisible by 5, so 0 or 5 must come in place of \$. But, a number ending with 5 is never divisible by 8. So, 0 will replace \$.

Now, the number formed by the last three digits is 4*0, which becomes divisible by 8, if * is replaced by 4.

Hence, digits in place of * and \$ are 4 and 0 respectively.

Ex. 17. Show that 4832718 is divisible by 11.

Sol. (Sum of digits at odd places) - (Sum of digits at even places)

$$= (8 + 7 + 3 + 4) - (1 + 2 + 8) = 11, \text{ which is divisible by 11.}$$

Hence, 4832718 is divisible by 11.

Ex. 18. Is 52563744 divisible by 24 ?

Sol. $24 = 3 \times 8$, where 3 and 8 are co-primes.

The sum of the digits in the given number is 36, which is divisible by 3. So, the given number is divisible by 3.

The number formed by the last 3 digits of the given number is 744, which is divisible by 8. So, the given number is divisible by 8.

Thus, the given number is divisible by both 3 and 8, where 3 and 8 are co-primes. So, it is divisible by 3×8 , i.e., 24.

Ex. 19. What least number must be added to 3000 to obtain a number exactly divisible by 19 ?

Sol. On dividing 3000 by 19, we get 17 as remainder.

$$\therefore \text{Number to be added} = (19 - 17) = 2.$$

Ex. 20. What least number must be subtracted from 2000 to get a number exactly divisible by 17 ?

Sol. On dividing 2000 by 17, we get 11 as remainder.

$$\therefore \text{Required number to be subtracted} = 11.$$

Ex. 21. Find the number which is nearest to 3105 and is exactly divisible by 21.

Sol. On dividing 3105 by 21, we get 18 as remainder.

$$\therefore \text{Number to be added to 3105} = (21 - 18) = 3.$$

$$\text{Hence, required number} = 3105 + 3 = 3108.$$

Ex. 22. Find the smallest number of 6 digits which is exactly divisible by 111.

Sol. Smallest number of 6 digits is 100000.

On dividing 100000 by 111, we get 100 as remainder.

$$\therefore \text{Number to be added} = (111 - 100) = 11.$$

$$\text{Hence, required number} = 100011.$$

Ex. 23. On dividing 15968 by a certain number, the quotient is 89 and the remainder is 37. Find the divisor.

$$\text{Sol. Divisor} = \frac{\text{Dividend} - \text{Remainder}}{\text{Quotient}} = \frac{15968 - 37}{89} = 179.$$

2. H.C.F. AND L.C.M. OF NUMBERS

IMPORTANT FACTS AND FORMULAE

- I. **Factors and Multiples** : If a number a divides another number b exactly, we say that a is a **factor** of b . In this case, b is called a **multiple** of a .
- II. **Highest Common Factor (H.C.F.) or Greatest Common Measure (G.C.M.) or Greatest Common Divisor (G.C.D.)** : The H.C.F. of two or more than two numbers is the greatest number that divides each of them exactly.
- There are two methods of finding the H.C.F. of a given set of numbers :
1. **Factorization Method** : Express each one of the given numbers as the product of prime factors. The product of least powers of common prime factors gives H.C.F.
 2. **Division Method** : Suppose we have to find the H.C.F. of two given numbers. Divide the larger number by the smaller one. Now, divide the divisor by the remainder. Repeat the process of dividing the preceding number by the remainder last obtained till zero is obtained as remainder. The last divisor is the required H.C.F.
- Finding the H.C.F. of more than two numbers** : Suppose we have to find the H.C.F. of three numbers. Then, H.C.F. of [(H.C.F. of any two) and (the third number)] gives the H.C.F. of three given numbers.
- Similarly, the H.C.F. of more than three numbers may be obtained.
- III. **Least Common Multiple (L.C.M.)** : The least number which is exactly divisible by each one of the given numbers is called their L.C.M.
1. **Factorization Method of Finding L.C.M.** : Resolve each one of the given numbers into a product of prime factors. Then, L.C.M. is the product of highest powers of all the factors.
 2. **Common Division Method (Short-cut Method) of Finding L.C.M.** : Arrange the given numbers in a row in any order. Divide by a number which divides exactly at least two of the given numbers and carry forward the numbers which are not divisible. Repeat the above process till no two of the numbers are divisible by the same number except 1. The product of the divisors and the undivided numbers is the required L.C.M. of the given numbers.
- IV. **Product of two numbers = Product of their H.C.F. and L.C.M.**
- V. **Co-primes** : Two numbers are said to be co-primes if their H.C.F. is 1.
- VI. **H.C.F. and L.C.M. of Fractions** :
- $$1. \text{ H.C.F.} = \frac{\text{H.C.F. of Numerators}}{\text{L.C.M. of Denominators}} \quad 2. \text{ L.C.M.} = \frac{\text{L.C.M. of Numerators}}{\text{H.C.F. of Denominators}}$$
- VII. **H.C.F. and L.C.M. of Decimal Fractions** : In given numbers, make the same number of decimal places by annexing zeros in some numbers, if necessary. Considering these numbers without decimal point, find H.C.F. or L.C.M. as the case may be. Now, in the result, mark off as many decimal places as are there in each of the given numbers.
- VIII. **Comparison of Fractions** : Find the L.C.M. of the denominators of the given fractions. Convert each of the fractions into an equivalent fraction with L.C.M. as the denominator, by multiplying both the numerator and denominator by the same number. The resultant fraction with the greatest numerator is the greatest.

SOLVED EXAMPLES

Ex. 1. Find the H.C.F. of $2^3 \times 3^2 \times 5 \times 7^4$, $2^2 \times 3^5 \times 5^2 \times 7^6$, $2^3 \times 5^3 \times 7^2$.

Sol. The prime numbers common to given numbers are 2, 5 and 7.

$$\therefore \text{H.C.F.} = 2^2 \times 5 \times 7^2 = 980.$$

Ex. 2. Find the H.C.F. of 108, 288 and 360.

Sol. $108 = 2^2 \times 3^3$, $288 = 2^5 \times 3^2$ and $360 = 2^3 \times 5 \times 3^2$.

$$\therefore \text{H.C.F.} = 2^2 \times 3^2 = 36.$$

Ex. 3. Find the H.C.F. of 513, 1134 and 1215.

$$\begin{array}{r} \text{Sol.} \quad 1134 \overline{) 1215} \quad 1 \\ \underline{1134} \\ 81 \end{array}$$

$$\begin{array}{r} 81 \overline{) 1134} \quad 14 \\ \underline{81} \\ 324 \\ \underline{324} \\ 0 \end{array}$$

\therefore H.C.F. of 1134 and 1215 is 81.

So, Required H.C.F. = H.C.F. of 513 and 81.

$$\begin{array}{r} 81 \overline{) 513} \quad 6 \\ \underline{486} \\ 27 \end{array}$$

$$\begin{array}{r} 27 \overline{) 81} \quad 3 \\ \underline{81} \\ 0 \end{array}$$

\therefore H.C.F. of given numbers = 27.

Ex. 4. Reduce $\frac{391}{667}$ to lowest terms.

Sol. H.C.F. of 391 and 667 is 23.

On dividing the numerator and denominator by 23, we get :

$$\frac{391}{667} = \frac{391 \div 23}{667 \div 23} = \frac{17}{29}.$$

Ex. 5. Find the L.C.M. of $2^2 \times 3^3 \times 5 \times 7^2$, $2^3 \times 3^2 \times 5^2 \times 7^4$, $2 \times 3 \times 5^3 \times 7 \times 11$.

Sol. L.C.M. = Product of highest powers of 2, 3, 5, 7 and 11 = $2^3 \times 3^3 \times 5^3 \times 7^4 \times 11$.

Ex. 6. Find the L.C.M. of 72, 108 and 2100.

Sol. $72 = 2^3 \times 3^2$, $108 = 3^3 \times 2^2$, $2100 = 2^2 \times 5^2 \times 3 \times 7$.

$$\therefore \text{L.C.M.} = 2^3 \times 3^3 \times 5^2 \times 7 = 37800.$$

Ex. 7. Find the L.C.M. of 16, 24, 36 and 54.

$$\begin{array}{r|rrrrr} \text{Sol.} & 2 & 16 & - & 24 & - & 36 & - & 54 \\ & 2 & 8 & - & 12 & - & 18 & - & 27 \\ & 2 & 4 & - & 6 & - & 9 & - & 27 \\ & 3 & 2 & - & 3 & - & 9 & - & 27 \\ & 3 & 2 & - & 1 & - & 3 & - & 9 \\ & & 2 & - & 1 & - & 1 & - & 3 \end{array}$$

$$\therefore \text{L.C.M.} = 2 \times 2 \times 2 \times 3 \times 3 \times 2 \times 3 = 432.$$

Ex. 8. Find the H.C.F. and L.C.M. of $\frac{2}{3}$, $\frac{8}{9}$, $\frac{16}{81}$ and $\frac{10}{27}$.

Sol. H.C.F. of given fractions = $\frac{\text{H.C.F. of } 2, 8, 16, 10}{\text{L.C.M. of } 3, 9, 81, 27} = \frac{2}{81}$

L.C.M. of given fractions = $\frac{\text{L.C.M. of } 2, 8, 16, 10}{\text{H.C.F. of } 3, 9, 81, 27} = \frac{80}{3}$

Ex. 9. Find the H.C.F. and L.C.M. of 0.63, 1.05 and 2.1.

Sol. Making the same number of decimal places, the given numbers are 0.63, 1.05 and 2.10.

Without decimal places, these numbers are 63, 105 and 210.

Now, H.C.F. of 63, 105 and 210 is 21.

\therefore H.C.F. of 0.63, 1.05 and 2.1 is 0.21.

L.C.M. of 63, 105 and 210 is 630.

\therefore L.C.M. of 0.63, 1.05 and 2.1 is 6.30.

Ex. 10. Two numbers are in the ratio of 15 : 11. If their H.C.F. is 13, find the numbers

Sol. Let the required numbers be $15x$ and $11x$.

Then, their H.C.F. is x . So, $x = 13$.

\therefore The numbers are (15×13) and (11×13) i.e., 195 and 143.

Ex. 11. The H.C.F. of two numbers is 11 and their L.C.M. is 693. If one of the numbers is 77, find the other.

Sol. Other number = $\left(\frac{11 \times 693}{77} \right) = 99$.

Ex. 12. Find the greatest possible length which can be used to measure exactly the lengths 4 m 95 cm, 9 m and 16 m 65 cm.

Sol. Required length = H.C.F. of 495 cm, 900 cm and 1665 cm.

$495 = 3^2 \times 5 \times 11$, $900 = 2^2 \times 3^2 \times 5^2$, $1665 = 3^2 \times 5 \times 37$.

\therefore H.C.F. = $3^2 \times 5 = 45$.

Hence, required length = 45 cm.

Ex. 13. Find the greatest number which on dividing 1657 and 2037 leaves remainders 5 and 5 respectively.

Sol. Required number = H.C.F. of $(1657 - 5)$ and $(2037 - 5)$ = H.C.F. of 1651 and 2032

$$\begin{array}{r}
 1651 \overline{) 2032} \quad 1 \\
 \underline{1651} \\
 381 \\
 1651 \overline{) 381} \quad 4 \\
 \underline{1524} \\
 127 \\
 1651 \overline{) 127} \quad 3 \\
 \underline{381} \\
 \times
 \end{array}$$

\therefore Required number = 127.

Ex. 14. Find the largest number which divides 62, 132 and 237 to leave the same remainder in each case.

Sol. Required number = H.C.F. of $(132 - 62)$, $(237 - 132)$ and $(237 - 62)$
 = H.C.F. of 70, 105 and 175 = 35.

3. DECIMAL FRACTIONS

IMPORTANT FACTS AND FORMULAE

I. Decimal Fractions : Fractions in which denominators are powers of 10 are known as **decimal fractions**.

Thus, $1/10 = 1 \text{ tenth} = .1$; $1/100 = 1 \text{ hundredth} = .01$;

$99/100 = 99 \text{ hundredths} = .99$; $7/1000 = 7 \text{ thousandths} = .007$, etc

II. Conversion of a Decimal Into Vulgar Fraction : Put 1 in the denominator under the decimal point and annex with it as many zeros as is the number of digits after the decimal point. Then remove the decimal point and reduce the fraction to its lowest terms.

Thus, $0.25 = 25/100 = 1/4$; $2.008 = 2008/1000 = 251/125$.

III. 1. Annexing zeros to the extreme right of a decimal fraction does not change its value

Thus, $0.8 = 0.80 = 0.800$, etc.

2. If numerator and denominator of a fraction contain the same number of decimal places, then we remove the decimal sign.

Thus, $1.84/2.99 = 184/299 = 8/13$; $0.365/0.584 = 365/584 = 5$

IV. Operations on Decimal Fractions :

1. Addition and Subtraction of Decimal Fractions : The given numbers are so placed under each other that the decimal points lie in one column. The numbers so arranged can now be added or subtracted in the usual way.

2. Multiplication of a Decimal Fraction By a Power of 10 : Shift the decimal point to the right by as many places as is the power of 10.

Thus, $5.9632 \times 100 = 596.32$; $0.073 \times 10000 = 0.0730 \times 10000 = 730$.

3. Multiplication of Decimal Fractions : Multiply the given numbers considering them without the decimal point. Now, in the product, the decimal point is marked off to obtain as many places of decimal as is the sum of the number of decimal places in the given numbers.

Suppose we have to find the product $(.2 \times .02 \times .002)$. Now, $2 \times 2 \times 2 = 8$. Sum of decimal places $= (1 + 2 + 3) = 6$. $.2 \times .02 \times .002 = .000008$.

4. Dividing a Decimal Fraction By a Counting Number : Divide the given number without considering the decimal point, by the given counting number. Now, in the quotient, put the decimal point to give as many places of decimal as there are in the dividend.

Suppose we have to find the quotient $(0.0204 \div 17)$. Now, $204 \div 17 = 12$. Dividend contains 4 places of decimal. So, $0.0204 \div 17 = 0.0012$.

5. Dividing a Decimal Fraction By a Decimal Fraction : Multiply both the dividend and the divisor by a suitable power of 10 to make divisor a whole number. Now, proceed as above.

Thus, $0.00066/0.11 = (0.00066*100)/(0.11*100) = (0.066/11) = 0.006V$

V. Comparison of Fractions : Suppose some fractions are to be arranged in ascending or descending order of magnitude. Then, convert each one of the given fractions in the decimal form, and arrange them accordingly.

Suppose, we have to arrange the fractions $3/5$, $6/7$ and $7/9$ in descending order.

now, $3/5=0.6, 6/7 = 0.857, 7/9 = 0.777....$

since $0.857>0.777...>0.6$, so $6/7>7/9>3/5$

VI. Recurring Decimal : If in a decimal fraction, a figure or a set of figures is repeated continuously, then such a number is called a *recurring decimal*.

In a recurring decimal, if a single figure is repeated, then it is expressed by putting a dot on it. If a set of figures is repeated, it is expressed by putting a bar on the set

Thus $1/3 = 0.3333.... = 0.\dot{3}$; $22/7 = 3.142857142857..... = 3.\overline{142857}$

Pure Recurring Decimal: A decimal fraction in which all the figures after the decimal point are repeated, is called a pure recurring decimal.

Converting a Pure Recurring Decimal Into Vulgar Fraction : Write the repeated figures only once in the numerator and take as many nines in the denominator as is the number of repeating figures.

thus $.0.\dot{5} = 5/9$; $0.\dot{5}3 = 53/99$; $0.0\dot{6}7 = 67/999$; etc...

Mixed Recurring Decimal: A decimal fraction in which some figures do not repeat and some of them are repeated, is called a mixed recurring decimal.

e.g., $0.17333... = 0.17\dot{3}$.

Converting a Mixed Recurring Decimal Into Vulgar Fraction : In the numerator, take the difference between the number formed by all the digits after decimal point (taking repeated digits only once) and that formed by the digits which are not repeated. In the denominator, take the number formed by as many nines as there are repeating digits followed by as many zeros as is the number of non-repeating digits.

Thus $0.1\dot{6} = (16-1) / 90 = 15/90 = 1/6$;

$0.\dot{2}273 = (2273 - 22)/9900 = 2251/9900$

VII. Some Basic Formulae :

1. $(a + b)(a - b) = (a^2 - b^2)$.
2. $(a + b)^2 = (a^2 + b^2 + 2ab)$.
3. $(a - b)^2 = (a^2 + b^2 - 2ab)$.
4. $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
5. $(a^3 + b^3) = (a + b) (a^2 - ab + b^2)$
6. $(a^3 - b^3) = (a - b) (a^2 + ab + b^2)$.
7. $(a^3 + b^3 + c^3 - 3abc) = (a + b + c) (a^2 + b^2 + c^2 - ab - bc - ca)$
8. When $a + b + c = 0$, then $a^3 + b^3 + c^3 = 3abc$

SOLVED EXAMPLES

Ex. 1. Convert the following into vulgar fraction:

- (i) 0.75 (ii) 3.004 (iii) 0.0056

Sol. (i). $0.75 = 75/100 = 3/4$ (ii) $3.004 = 3004/1000 = 751/250$ (iii) $0.0056 = 56/10000 = 7/1250$

Ex. 2. Arrange the fractions $5/8$, $7/12$, $13/16$, $16/29$ and $3/4$ in ascending order of magnitude.

Sol. Converting each of the given fractions into decimal form, we get :

$$5/8 = 0.624, 7/12 = 0.5833, 16/29 = 0.5517, \text{ and } 3/4 = 0.75$$

$$\text{Now, } 0.5517 < 0.5833 < 0.625 < 0.75 < 0.8125$$

$$\therefore 16/29 < 7/12 < 5/8 < 3/4 < 13/16$$

Ex. 3. arrange the fractions $3/5$, $4/7$, $8/9$, and $9/11$ in their descending order.

Sol. Clearly, $3/5 = 0.6$, $4/7 = 0.571$, $8/9 = 0.88$, $9/11 = 0.818$.

$$\text{Now, } 0.88 > 0.818 > 0.6 > 0.571$$

$$\therefore 8/9 > 9/11 > 3/5 > 4/7$$

Ex. 4. Evaluate : (i) $6202.5 + 620.25 + 62.025 + 6.2025 + 0.62025$

(ii) $5.064 + 3.98 + 0.7036 + 7.6 + 0.3 + 2$

Sol. (i) 6202.5

$$620.25$$

$$62.025$$

$$6.2025$$

$$+ \underline{0.62025}$$

$$\underline{6891.59775}$$

(ii) 5.064

$$3.98$$

$$0.7036$$

$$7.6$$

$$0.3$$

$$\underline{2.0}$$

$$\underline{19.6476}$$

Ex. 5. Evaluate : (i) $31.004 - 17.2368$

(ii) $13 - 5.1967$

Sol. (i)
$$\begin{array}{r} 31.0040 \\ - 17.2386 \\ \hline 13.7654 \end{array}$$

(ii)
$$\begin{array}{r} 31.0000 \\ - 5.1967 \\ \hline 25.8033 \end{array}$$

Ex. 6. What value will replace the question mark in the following equations ?

(i) $5172.49 + 378.352 + ? = 9318.678$

(ii) $? - 7328.96 + 5169.38$

Sol. (i) Let $5172.49 + 378.352 + x = 9318.678$

Then, $x = 9318.678 - (5172.49 + 378.352) = 9318.678 - 5550.842 =$

(ii) Let $x - 7328.96 = 5169.38$. Then, $x = 5169.38 + 7328.96 = 12498.34$.

Ex. 7. Find the products: (i) 6.3204×100

(ii) 0.069×10000

Sol. (i) $6.3204 \times 1000 = 632.04$

(ii) $0.069 \times 10000 = 0.0690 \times 10000 = 690$

Ex. 8. Find the product:

(i) 2.61×1.3

(ii) 2.1693×1.4

(iii) $0.4 \times 0.04 \times 0.004 \times 40$

Sol. (i) $261 \times 13 = 3393$. Sum of decimal places of given numbers $= (2+1) = 3$.

$2.61 \times 1.3 = 3.393$.

(ii) $21693 \times 14 = 303702$. Sum of decimal places $= (4+1) = 5$

$2.1693 \times 1.4 = 3.03702$.

(iii) $4 \times 4 \times 4 \times 40 = 2560$. Sum of decimal places $= (1 + 2 + 3) = 6$

$0.4 \times 0.04 \times 0.004 \times 40 = 0.002560$.

Ex. 9. Given that $268 \times 74 = 19832$, find the values of 2.68×0.74 .

Sol. Sum of decimal places $= (2 + 2) = 4$

$2.68 \times 0.74 = 1.9832$.

Ex. 10. Find the quotient:

(i) $0.63 \div 9$

(ii) $0.0204 \div 17$

(iii) $3.1603 \div 13$

Sol. (i) $63 \div 9 = 7$. Dividend contains 2 places decimal.

$0.63 \div 9 = 0.07$.

(ii) $204 \div 17 = 12$. Dividend contains 4 places of decimal.

$0.0204 \div 17 = 0.0012$.

(iii) $31603 \div 13 = 2431$. Dividend contains 4 places of decimal.

$3.1603 \div 13 = 0.2431$.

Ex. 11. Evaluate :

(i) $35 + 0.07$

(ii) $2.5 + 0.0005$

(iii) $136.09 + 43.9$

Sol. (i) $35/0.07 = (35 \times 100) / (0.07 \times 100) = (3500 / 7) = 500$
(ii) $25/0.0005 = (25 \times 10000) / (0.0005 \times 10000) = 25000 / 5 = 5000$
(iii) $136.09/43.9 = (136.09 \times 10) / (43.9 \times 10) = 1360.9 / 439 = 3.1$

Ex. 12. What value will come in place of question mark in the following equation?

(i) $0.006 + ? = 0.6$ (ii) $? + 0.025 = 80$

Sol. (i) Let $0.006 / x = 0.6$, Then, $x = (0.006 / 0.6) = (0.006 \times 10) / (0.6 \times 10) = 0.06/6 = 0.01$
(ii) Let $x / 0.025 = 80$, Then, $x = 80 \times 0.025 = 2$

Ex. 13. If $(1 / 3.718) = 0.2689$, Then find the value of $(1 / 0.0003718)$.

Sol. $(1 / 0.0003718) = (10000 / 3.718) = 10000 \times (1 / 3.718) = 10000 \times 0.2689 = 2689$.

Ex. 14. Express as vulgar fractions : (i) $0.\overline{37}$ (ii) $0.\overline{053}$ (iii) $3.\overline{142857}$

Sol. (i) $0.\overline{37} = 37 / 99$, (ii) $0.\overline{053} = 53 / 999$

(iii) $3.\overline{142857} = 3 + 0.\overline{142857} = 3 + (142857 / 999999) = 3 (142857/999999)$

Ex. 15. Express as vulgar fractions : (i) $0.\overline{17}$ (ii) $0.\overline{1254}$ (iii) $2.\overline{536}$

Sol. (i) $0.\overline{17} = (17 - 1)/90 = 16 / 90 = 8 / 45$

(ii) $0.\overline{1254} = (1254 - 12) / 9900 = 1242 / 9900 = 69 / 550$

(iii) $2.\overline{536} = 2 + 0.\overline{536} = 2 + (536 - 53)/900 = 2 + (483/900) = 2 + (161/300) = 2 (161/300)$

Ex. 16. Simplify: $\frac{0.05 \times 0.05 \times 0.05 + 0.04 \times 0.04 \times 0.04}{0.05 \times 0.05 - 0.05 \times 0.04 + 0.04 \times 0.04}$

Sol. Given expression $= (a^3 + b^3) / (a^2 - ab + b^2)$, where $a = 0.05$, $b = 0.04$
 $= (a + b) = (0.05 + 0.04) = 0.09$

ALERT FOR GOVERNMENT JOBS & UPSC JOB TEAM

4. SIMPLIFICATION

IMPORTANT CONCEPTS

- I. **'BODMAS' Rule** : This rule depicts the correct sequence in which the operations are to be executed, so as to find out the value of a given expression.

Here, '**B**' stands for '**Bracket**', '**O**' for '**of**', '**D**' for '**Division**', '**M**' for '**Multiplication**', '**A**' for '**Addition**' and '**S**' for '**Subtraction**'.

Thus, in simplifying an expression, first of all the brackets must be removed, strictly in the order $()$, $\{\}$ and $[\]$.

After removing the brackets, we must use the following operations strictly in the order :

(i) of (ii) Division (iii) Multiplication (iv) Addition (v) Subtraction.

- II. **Modulus of a Real Number** : Modulus of a real number a is defined as

$$|a| = \begin{cases} a, & \text{if } a > 0 \\ -a, & \text{if } a < 0. \end{cases}$$

Thus, $|5| = 5$ and $|-5| = -(-5) = 5$.

- III. **Virnaculum (or Bar)** : When an expression contains Virnaculum, before applying the 'BODMAS' rule, we simplify the expression under the Virnaculum.

SOLVED EXAMPLES

- Ex. 1. Simplify :** (i) $5005 - 5000 + 10$ (ii) $18800 \div 470 + 20$.

Sol. (i) $5005 - 5000 + 10 = 5005 - \frac{5000}{10} = 5005 - 500 = 4505$.

(ii) $18800 \div 470 + 20 = \frac{18800}{470} + 20 = 40 + 20 = 60$.

- Ex. 2. Simplify :** $b - [b - (a + b) - \{b - (b - a - b)\} + 2a]$. (Hotel Management, 2002)

Sol. Given expression $= b - [b - (a + b) - \{b - (b - a - b)\} + 2a]$

$$= b - [b - a - b - \{b - 2b + a\} + 2a]$$

$$= b - [-a - \{b - 2b + a + 2a\}]$$

$$= b - [-a - \{-b + 3a\}] = b - [-a + b - 3a]$$

$$= b - [-4a + b] = b + 4a - b = 4a.$$

- Ex. 3. What value will replace the question mark in the following equation ?**

$$4\frac{1}{2} + 3\frac{1}{6} + ? + 2\frac{1}{3} = 13\frac{2}{5}.$$

Sol. Let $\frac{9}{2} + \frac{19}{6} + x + \frac{7}{3} = \frac{67}{5}$.

$$\text{Then, } x = \frac{67}{5} - \left(\frac{9}{2} + \frac{19}{6} + \frac{7}{3}\right) \Leftrightarrow x = \frac{67}{5} - \left(\frac{27 + 19 + 14}{6}\right) = \left(\frac{67}{5} - \frac{60}{6}\right)$$

$$\Leftrightarrow x = \left(\frac{67}{5} - 10\right) = \frac{17}{5} = 3\frac{2}{5}.$$

Hence, missing fraction $= 3\frac{2}{5}$.

Ex. 4. $\frac{4}{15}$ of $\frac{5}{7}$ of a number is greater than $\frac{4}{9}$ of $\frac{2}{5}$ of the same number by 8.
What is half of that number ? (S.B.I.P.O. 2000)

Sol. Let the number be x . Then, $\frac{4}{15}$ of $\frac{5}{7}$ of $x - \frac{4}{9}$ of $\frac{2}{5}$ of $x = 8 \Leftrightarrow \frac{4}{21}x - \frac{8}{45}x = 8$
 $\Leftrightarrow \left(\frac{4}{21} - \frac{8}{45}\right)x = 8 \Leftrightarrow \left(\frac{60 - 56}{315}\right)x = 8 \Leftrightarrow \frac{4}{315}x = 8$
 $\Leftrightarrow x = \left(\frac{8 \times 315}{4}\right) = 630 \Leftrightarrow \frac{1}{2}x = 315.$

Hence, required number = 315.

Ex. 5. Simplify : $3\frac{1}{4} \div \left\{ 1\frac{1}{4} - \frac{1}{2} \left(2\frac{1}{2} - \frac{1}{4} - \frac{1}{6} \right) \right\}.$

Sol. Given exp. = $\left[\frac{13}{4} \div \left\{ \frac{5}{4} - \frac{1}{2} \left(\frac{5}{2} - \frac{3}{12} - \frac{1}{6} \right) \right\} \right] = \left[\frac{13}{4} \div \left\{ \frac{5}{4} - \frac{1}{2} \left(\frac{5}{2} - \frac{1}{2} \right) \right\} \right]$
 $= \left[\frac{13}{4} \div \left\{ \frac{5}{4} - \frac{1}{2} \left(\frac{30 - 1}{12} \right) \right\} \right] = \left[\frac{13}{4} \div \left\{ \frac{5}{4} - \frac{29}{24} \right\} \right]$
 $= \left[\frac{13}{4} \div \left\{ \frac{30 - 29}{24} \right\} \right] = \left[\frac{13}{4} \div \frac{1}{24} \right] = \left[\frac{13}{4} \times 24 \right] = 78.$

Ex. 6. Simplify : $108 \div 36$ of $\frac{1}{4} + \frac{2}{5} \times 3\frac{1}{4}.$

Sol. Given exp. = $108 \div 9 + \frac{2}{5} \times \frac{13}{4} = \frac{108}{9} + \frac{13}{10} = \left(12 + \frac{13}{10} \right) = \frac{133}{10} = 13\frac{3}{10}.$

Ex. 7. Simplify : $\frac{7}{2} \div \frac{5}{2} \times \frac{3}{2} \div 5.25.$ (S.S.C. 1999)

Sol. Given exp. = $\frac{7}{2} \times \frac{2}{5} \times \frac{3}{2} \div 5.25 = \frac{21}{10} \div \frac{525}{100} = \frac{21}{10} \times \frac{15}{14} \times \frac{100}{525} = \frac{6}{14} = \frac{3}{7}.$

Ex. 8. Simplify : (i) $12.05 \times 5.4 \div 0.6$ (ii) $.6 \times .6 + .6 \div 6.$ (Bank P.O. 2003)

Sol. (i) Given exp. = $12.05 \times \frac{5.4}{0.6} = 12.05 \times 9 = 108.45.$

(ii) Given exp. = $.6 \times .6 + \frac{.6}{6} = .36 + .1 = .46.$

Ex. 9. Find the value of x in each of the following equations :

(i) $\frac{17.28 \div x}{3.6 \times 0.2} = 2$ (ii) $3648.24 \div 364.824 + x - 36.4824 = 3794.1696$

(iii) $8.5 - \left\{ 5\frac{1}{2} - \left(7\frac{1}{2} + 2.8 \div x \right) \right\} \times 4.25 \div (0.2)^2 = 306.$ (Hotel Management, 1997)

Sol. (i) $\frac{17.28}{x} = 2 \times 3.6 \times 0.2 \Leftrightarrow x = \frac{17.28}{1.44} = \frac{1728}{144} = 12.$

$$(ii) \frac{364.824}{x} = (3794.1696 + 36.4824) - 3648.24 = 3830.652 - 3648.24 = 182.412$$

$$\Leftrightarrow x = \frac{364.824}{182.412} = 2$$

$$(iii) 8.5 - \left\{ 5.5 - \left(7.5 + \frac{2.8}{x} \right) \right\} \times \frac{4.25}{0.04} = 306 \Leftrightarrow 8.5 - \left\{ 5.5 - \left(\frac{7.5x + 2.8}{x} \right) \right\} \times \frac{4.25}{4} = 306$$

$$\Leftrightarrow 8.5 - \left\{ \frac{5.5x - 7.5x - 2.8}{x} \right\} \times \frac{4.25}{4} = 306 \Leftrightarrow 8.5 - \left\{ \frac{-2x - 2.8}{x} \right\} \times 106.25 = 306$$

$$\Leftrightarrow 8.5 - \left\{ \frac{-212.5x - 297.5}{x} \right\} = 306 \Leftrightarrow \frac{8.5x + 212.5x + 297.5}{x} = 306$$

$$\Leftrightarrow (306 - 221)x = 297.5 \Leftrightarrow x = \frac{297.5}{85} = 3.5$$

Ex. 10. If $\frac{x}{y} = \frac{6}{5}$, find the value of $\frac{x^2 + y^2}{x^2 - y^2}$.

$$\text{Sol. } \frac{x^2 + y^2}{x^2 - y^2} = \frac{\frac{x^2}{y^2} + 1}{\frac{x^2}{y^2} - 1} = \frac{\left(\frac{x}{y}\right)^2 + 1}{\left(\frac{x}{y}\right)^2 - 1} = \frac{\left(\frac{6}{5}\right)^2 + 1}{\left(\frac{6}{5}\right)^2 - 1} = \frac{\frac{36}{25} + 1}{\frac{36}{25} - 1} = \frac{61}{25} \times \frac{25}{11} = \frac{61}{11}$$

ALERT FOR GOVERNMENT JOBS & UPSC JOB TEAM

5. SQUARE ROOTS AND CUBE ROOTS

IMPORTANT FACTS AND FORMULAE

Square Root : If $x^2 = y$, we say that the square root of y is x and we write, $\sqrt{y} = x$.

Thus, $\sqrt{4} = 2$, $\sqrt{9} = 3$, $\sqrt{196} = 14$.

Cube Root : The cube root of a given number x is the number whose cube is x . We denote the cube root of x by $\sqrt[3]{x}$.

Thus, $\sqrt[3]{8} = \sqrt[3]{2 \times 2 \times 2} = 2$, $\sqrt[3]{343} = \sqrt[3]{7 \times 7 \times 7} = 7$ etc.

Note :

$$1. \sqrt{xy} = \sqrt{x} \times \sqrt{y} \qquad 2. \sqrt{\frac{x}{y}} = \frac{\sqrt{x}}{\sqrt{y}} = \frac{\sqrt{x}}{\sqrt{y}} \times \frac{\sqrt{y}}{\sqrt{y}} = \frac{\sqrt{xy}}{y}$$

SOLVED EXAMPLES

Ex. 1. Evaluate $\sqrt{6084}$ by factorization method.

Sol. Method : Express the given number as the product of prime factors.
Now, take the product of these prime factors choosing one out of every pair of the same primes. This product gives the square root of the given number.

Thus, resolving 6084 into prime factors, we get :

$$6084 = 2^2 \times 3^2 \times 13^2$$

$$\therefore \sqrt{6084} = (2 \times 3 \times 13) = 78.$$

2	6084
2	3042
3	1521
3	507
13	169
	13

Ex. 2. Find the square root of 1471369.

Sol. Explanation : In the given number, mark off the digits in pairs starting from the unit's digit. Each pair and the remaining one digit is called a period.

Now, $1^2 = 1$. On subtracting, we get 0 as remainder.

Now, bring down the next period i.e., 47.

Now, trial divisor is $1 \times 2 = 2$ and trial dividend is 47.

So, we take 22 as divisor and put 2 as quotient.

The remainder is 3.

Next, we bring down the next period which is 13.

Now, trial divisor is $12 \times 2 = 24$ and trial dividend is 313. So, we take 241 as dividend and 1 as quotient.

The remainder is 72.

Bring down the next period i.e., 69.

Now, the trial divisor is $121 \times 2 = 242$ and the trial dividend is 7269. So, we take 3 as quotient and 2423 as divisor. The remainder is then zero.

$$\text{Hence, } \sqrt{1471369} = 1213.$$

1	1471369 (1213
	1
22	47
	44
241	313
	241
2423	7269
	7269
	x

Ex. 3. Evaluate : $\sqrt{248 + \sqrt{51 + \sqrt{169}}}$.

Sol. Given expression = $\sqrt{248 + \sqrt{51 + 13}} = \sqrt{248 + \sqrt{64}} = \sqrt{248 + 8} = \sqrt{256} = 16$.

Ex. 4. If $a * b * c = \frac{\sqrt{(a+2)(b+3)}}{c+1}$, then find the value of $6 * 15 * 3$.

Sol. $6 * 15 * 3 = \frac{\sqrt{(6+2)(15+3)}}{3+1} = \frac{\sqrt{8 \times 18}}{4} = \frac{\sqrt{144}}{4} = \frac{12}{4} = 3$.

Ex. 5. Find the value of $\sqrt{1\frac{9}{16}}$.

Sol. $\sqrt{1\frac{9}{16}} = \sqrt{\frac{25}{16}} = \frac{\sqrt{25}}{\sqrt{16}} = \frac{5}{4} = 1\frac{1}{4}$.

Ex. 6. What is the square root of 0.0009 ?

Sol. $\sqrt{0.0009} = \sqrt{\frac{9}{10000}} = \frac{\sqrt{9}}{\sqrt{10000}} = \frac{3}{100} = 0.03$.

Ex. 7. Evaluate $\sqrt{175.2976}$.

Sol. Method : We make even number of decimal places by affixing a zero, if necessary. Now, we mark off periods and extract the square root as shown.

$\therefore \sqrt{175.2976} = 13.24$.

1	$\overline{175.2976}$ (13.24
1	
23	$\overline{75}$
	$\underline{69}$
262	$\overline{629}$
	$\underline{524}$
2644	$\overline{10576}$
	$\underline{10576}$
	\times

Ex. 8. What will come in place of question mark in each of the following questions?

(i) $\sqrt{\frac{32.4}{?}} = 2$

(ii) $\sqrt{86.49} + \sqrt{5 + (?)^2} = 12.3$. (R.R.B. 2002)

Sol. (i) Let $\sqrt{\frac{32.4}{x}} = 2$. Then, $\frac{32.4}{x} = 4 \Leftrightarrow 4x = 32.4 \Leftrightarrow x = 8.1$.

(ii) Let $\sqrt{86.49} + \sqrt{5 + x^2} = 12.3$.

Then, $9.3 + \sqrt{5 + x^2} = 12.3 \Leftrightarrow \sqrt{5 + x^2} = 12.3 - 9.3 = 3$

$\Leftrightarrow 5 + x^2 = 9 \Leftrightarrow x^2 = 9 - 5 = 4 \Leftrightarrow x = \sqrt{4} = 2$.

Ex. 9. Find the value of $\sqrt{\frac{0.289}{0.00121}}$. (IGNOU, 2003)

Sol. $\sqrt{\frac{0.289}{0.00121}} = \sqrt{\frac{0.28900}{0.00121}} = \sqrt{\frac{28900}{121}} = \frac{170}{11}$.

Ex. 10. If $\sqrt{1 + \frac{x}{144}} = \frac{13}{12}$, then find the value of x .

Sol. $\sqrt{1 + \frac{x}{144}} = \frac{13}{12} \Rightarrow \left(1 + \frac{x}{144}\right) = \left(\frac{13}{12}\right)^2 = \frac{169}{144} \Rightarrow \frac{x}{144} = \frac{169}{144} - 1$
 $\Rightarrow \frac{x}{144} = \frac{25}{144} \Rightarrow x = 25.$

Ex. 11. Find the value of $\sqrt{3}$ upto three places of decimal.

Sol.

1	3.000000 (1.732
	1
27	200
	189
343	1100
	1029
3462	7100
	6924

$\therefore \sqrt{3} = 1.732.$

Ex. 12. If $\sqrt{3} = 1.732$, find the value of $\sqrt{192} - \frac{1}{2}\sqrt{48} - \sqrt{75}$ correct to 3 places decimal. (S.S.C. 2004)

Sol. $\sqrt{192} - \frac{1}{2}\sqrt{48} - \sqrt{75} = \sqrt{64 \times 3} - \frac{1}{2}\sqrt{16 \times 3} - \sqrt{25 \times 3} = 8\sqrt{3} - \frac{1}{2} \times 4\sqrt{3} - 5\sqrt{3}$
 $= 3\sqrt{3} - 2\sqrt{3} = \sqrt{3} = 1.732$

Ex. 13. Evaluate : $\sqrt{\frac{9.5 \times .0085 \times 18.9}{.0017 \times 1.9 \times 0.021}}$

Sol. Given exp. = $\sqrt{\frac{9.5 \times .0085 \times 18.900}{.0017 \times 1.9 \times 0.021}}$

Now, since the sum of decimal places in the numerator and denominator under the radical sign is the same, we remove the decimal.

\therefore Given exp. = $\sqrt{\frac{95 \times 85 \times 18900}{17 \times 19 \times 21}} = \sqrt{5 \times 5 \times 900} = 5 \times 30 = 150.$

Ex. 14. Simplify : $\sqrt{[(12.1)^2 - (8.1)^2] \div [(0.25)^2 + (0.25)(19.95)]}$. (C.B.I. 2003)

Sol. Given exp. = $\sqrt{\frac{(12.1 + 8.1)(12.1 - 8.1)}{(0.25)(0.25 + 19.95)}} = \sqrt{\frac{20.2 \times 4}{0.25 \times 20.2}}$
 $= \sqrt{\frac{4}{0.25}} = \sqrt{\frac{400}{25}} = \sqrt{16} = 4.$

Ex. 15. If $x = 1 + \sqrt{2}$ and $y = 1 - \sqrt{2}$, find the value of $(x^2 + y^2)$.

Sol. $x^2 + y^2 = (1 + \sqrt{2})^2 + (1 - \sqrt{2})^2 = 2[(1)^2 + (\sqrt{2})^2] = 2 \times 3 = 6.$

Ex. 16. Evaluate $\sqrt{0.9}$ upto 3 places of decimal. (R.R.B. 2003)

Sol.

9	0.900000 (.948
	81
184	900
	736
1888	16400
	15104

$\therefore \sqrt{0.9} = 0.948.$

6. AVERAGE

IMPORTANT FACTS AND FORMULAE

1. $\text{Average} = \left(\frac{\text{Sum of observations}}{\text{Number of observations}} \right)$.
2. Suppose a man covers a certain distance at x kmph and an equal distance at y kmph. Then, the average speed during the whole journey is $\left(\frac{2xy}{x+y} \right)$ kmph.

SOLVED EXAMPLES

Ex. 1. Find the average of all prime numbers between 30 and 50.

Sol. There are five prime numbers between 30 and 50.

They are 31, 37, 41, 43 and 47.

$$\therefore \text{Required average} = \left(\frac{31 + 37 + 41 + 43 + 47}{5} \right) = \frac{199}{5} = 39.8.$$

Ex. 2. Find the average of first 40 natural numbers.

$$\text{Sol. Sum of first } n \text{ natural numbers} = \frac{n(n+1)}{2}.$$

$$\text{So, sum of first 40 natural numbers} = \frac{40 \times 41}{2} = 820.$$

$$\therefore \text{Required average} = \frac{820}{40} = 20.5.$$

Ex. 3. Find the average of first 20 multiples of 7.

$$\text{Sol. Required average} = \frac{7(1+2+3+\dots+20)}{20} = \left(\frac{7 \times 20 \times 21}{20 \times 2} \right) = \left(\frac{147}{2} \right) = 73.5.$$

Ex. 4. The average of four consecutive even numbers is 27. Find the largest of these numbers.

Sol. Let the numbers be x , $x+2$, $x+4$ and $x+6$. Then,

$$\frac{x + (x+2) + (x+4) + (x+6)}{4} = 27 \Rightarrow \frac{4x+12}{4} = 27 \Rightarrow x+3 = 27 \Rightarrow x = 24.$$

$$\therefore \text{Largest number} = (x+6) = 24+6 = 30.$$

Ex. 5. There are two sections A and B of a class, consisting of 36 and 44 students respectively. If the average weight of section A is 40 kg and that of section B is 35 kg, find the average weight of the whole class.

Sol. Total weight of $(36 + 44)$ students $= (36 \times 40 + 44 \times 35)$ kg $= 2980$ kg.

$$\therefore \text{Average weight of the whole class} = \left(\frac{2980}{80} \right) \text{ kg} = 37.25 \text{ kg}.$$

Ex. 6. Nine persons went to a hotel for taking their meals. Eight of them spent Rs. 12 each on their meals and the ninth spent Rs. 8 more than the average expenditure of all the nine. What was the total money spent by them?

Sol. Let the average expenditure of all the nine be Rs. x .

Then, $12 \times 8 + (x + 8) = 9x$ or $8x = 104$ or $x = 13$.

\therefore Total money spent = $9x = \text{Rs. } (9 \times 13) = \text{Rs. } 117$.

Ex. 7. Of the three numbers, second is twice the first and is also thrice the third. If the average of the three numbers is 44, find the largest number.

Sol. Let the third number be x . Then, second number = $3x$. First number = $\frac{3x}{2}$.

$\therefore x + 3x + \frac{3x}{2} = (44 \times 3)$ or $\frac{11x}{2} = 44 \times 3$ or $x = 24$.

So, largest number = 2nd number = $3x = 72$.

Ex. 8. The average of 25 results is 18. The average of first twelve of them is 14 and that of last twelve is 17. Find the thirteenth result.

Sol. Clearly, thirteenth result = (sum of 25 results) - (sum of 24 results)

$$= (18 \times 25) - [(14 \times 12) + (17 \times 12)]$$

$$= 450 - (168 + 204) = 450 - 372 = 78.$$

Ex. 9. The average of 11 results is 60. If the average of first six results is 58 and that of the last six is 63, find the sixth result.

Sol. Sixth result = $(58 \times 6 + 63 \times 6 - 60 \times 11) = 66$.

Ex. 10. The average weight of A, B, C is 45 kg. If the average weight of A and B be 40 kg and that of B and C be 43 kg, find the weight of B.

Sol. Let A, B and C represent their individual weights. Then,

$$A + B + C = (45 \times 3) \text{ kg} = 135 \text{ kg.}$$

$$A + B = (40 \times 2) \text{ kg} = 80 \text{ kg and } B + C = (43 \times 2) \text{ kg} = 86 \text{ kg.}$$

$$\therefore B = (A + B) + (B + C) - (A + B + C) = (80 + 86 - 135) \text{ kg} = 31 \text{ kg.}$$

Ex. 11. The average age of a class of 39 students is 15 years. If the age of the teacher be included, then the average increases by 3 months. Find the age of the teacher.

Sol. Total age of 39 persons = (39×15) years = 585 years.

$$\text{Average age of 40 persons} = 15 \text{ years } 3 \text{ months} = \frac{61}{4} \text{ years.}$$

$$\text{Total age of 40 persons} = \left(\frac{61}{4} \times 40 \right) \text{ years} = 610 \text{ years.}$$

$$\therefore \text{Age of the teacher} = (610 - 585) \text{ years} = 25 \text{ years.}$$

Ex. 12. The average weight of 10 oarsmen in a boat is increased by 1.8 kg when one of the crew, who weighs 53 kg is replaced by a new man. Find the weight of the new man.

Sol. Total weight increased = $(1.8 \times 10) \text{ kg} = 18 \text{ kg.}$

$$\therefore \text{Weight of the new man} = (53 + 18) \text{ kg} = 71 \text{ kg.}$$

Ex. 13. There were 35 students in a hostel. Due to the admission of 7 new students, the expenses of the mess were increased by Rs. 42 per day while the average expenditure per head diminished by Re 1. What was the original expenditure of the mess?

Sol. Let the original average expenditure be Rs. x . Then,

$$42(x - 1) - 35x = 42 \Leftrightarrow 7x = 84 \Rightarrow x = 12.$$

$$\therefore \text{Original expenditure} = \text{Rs. } (35 \times 12) = \text{Rs. } 420.$$

Ex. 14. A batsman makes a score of 87 runs in the 17th inning and thus increases his average by 3. Find his average after 17th inning.

Sol. Let the average after 17th inning = x

Then, average after 16th inning = $(x - 3)$.

$$\therefore 16(x - 3) + 87 = 17x \text{ or } x = (87 - 48) = 39.$$

Ex. 15. Distance between two stations A and B is 778 km. A train covers the journey from A to B at 84 km per hour and returns back to A with a uniform speed of 56 km per hour. Find the average speed of the train during the whole journey.

Sol. Required average speed = $\left(\frac{2xy}{x + y} \right) \text{ km/hr} = \frac{2 \times 84 \times 56}{(84 + 56)} \text{ km/hr}$

$$= \left(\frac{2 \times 84 \times 56}{140} \right) \text{ km/hr} = 67.2 \text{ km/hr.}$$

ALERT FOR GOVERNMENT JOBS & UPSC JOB TEAM

7. PROBLEMS ON NUMBERS

In this section, questions involving a set of numbers are put in the form of a puzzle. You have to analyse the given conditions, assume the unknown numbers and form equations accordingly, which on solving yield the unknown numbers.

SOLVED EXAMPLES

Ex. 1. A number is as much greater than 36 as is less than 86. Find the number.

Sol. Let the number be x . Then, $x - 36 = 86 - x \Rightarrow 2x = 86 + 36 = 122 \Rightarrow x = 61$.
Hence, the required number is 61.

Ex. 2. Find a number such that when 15 is subtracted from 7 times the number, the result is 10 more than twice the number. (Hotel Management, 2002)

Sol. Let the number be x . Then, $7x - 15 = 2x + 10 \Rightarrow 5x = 25 \Rightarrow x = 5$.
Hence, the required number is 5.

Ex. 3. The sum of a rational number and its reciprocal is $\frac{13}{6}$. Find the number. (S.S.C. 2000)

Sol. Let the number be x .

$$\begin{aligned} \text{Then, } x + \frac{1}{x} &= \frac{13}{6} \Rightarrow \frac{x^2 + 1}{x} = \frac{13}{6} \Rightarrow 6x^2 - 13x + 6 = 0 \\ &\Rightarrow 6x^2 - 9x - 4x + 6 = 0 \Rightarrow (3x - 2)(2x - 3) = 0 \\ &\Rightarrow x = \frac{2}{3} \text{ or } x = \frac{3}{2}. \end{aligned}$$

Hence, the required number is $\frac{2}{3}$ or $\frac{3}{2}$.

Ex. 4. The sum of two numbers is 184. If one-third of the one exceeds one-seventh of the other by 8, find the smaller number.

Sol. Let the numbers be x and $(184 - x)$. Then,

$$\frac{x}{3} - \frac{(184 - x)}{7} = 8 \Rightarrow 7x - 3(184 - x) = 168 \Rightarrow 10x = 720 \Rightarrow x = 72.$$

So, the numbers are 72 and 112. Hence, smaller number = 72.

Ex. 5. The difference of two numbers is 11 and one-fifth of their sum is 9. Find the numbers.

Sol. Let the numbers be x and y . Then,

$$x - y = 11 \quad \dots(i) \quad \text{and} \quad \frac{1}{5}(x + y) = 9 \Rightarrow x + y = 45 \quad \dots(ii)$$

Adding (i) and (ii), we get : $2x = 56$ or $x = 28$. Putting $x = 28$ in (i), we get : $y = 17$.
Hence, the numbers are 28 and 17.

Ex. 6. If the sum of two numbers is 42 and their product is 437, then find the absolute difference between the numbers. (S.S.C. 2003)

Sol. Let the numbers be x and y . Then, $x + y = 42$ and $xy = 437$.

$$x - y = \sqrt{(x + y)^2 - 4xy} = \sqrt{(42)^2 - 4 \times 437} = \sqrt{1764 - 1748} = \sqrt{16} = 4.$$

\therefore Required difference = 4.

Ex. 7. The sum of two numbers is 15 and the sum of their squares is 113. Find the numbers.

Sol. Let the numbers be x and $(15 - x)$.

$$\text{Then, } x^2 + (15 - x)^2 = 113 \Leftrightarrow x^2 + 225 + x^2 - 30x = 113$$

$$\Leftrightarrow 2x^2 - 30x + 112 = 0 \Leftrightarrow x^2 - 15x + 56 = 0$$

$$\Leftrightarrow (x - 7)(x - 8) = 0 \Leftrightarrow x = 7 \text{ or } x = 8.$$

So, the numbers are 7 and 8.

Ex. 8. The average of four consecutive even numbers is 27. Find the largest of these numbers.

Sol. Let the four consecutive even numbers be $x, x + 2, x + 4$ and $x + 6$.

$$\text{Then, sum of these numbers} = (27 \times 4) = 108.$$

$$\text{So, } x + (x + 2) + (x + 4) + (x + 6) = 108 \text{ or } 4x = 96 \text{ or } x = 24.$$

$$\therefore \text{Largest number} = (x + 6) = 30.$$

Ex. 9. The sum of the squares of three consecutive odd numbers is 2531. Find the numbers.

Sol. Let the numbers be $x, x + 2$ and $x + 4$.

$$\text{Then, } x^2 + (x + 2)^2 + (x + 4)^2 = 2531 \Leftrightarrow 3x^2 + 12x - 2511 = 0$$

$$\Leftrightarrow x^2 + 4x - 837 = 0 \Leftrightarrow (x - 27)(x + 31) = 0 \Leftrightarrow x = 27.$$

Hence, the required numbers are 27, 29 and 31.

Ex. 10. Of two numbers, 4 times the smaller one is less than 3 times the larger one by 5. If the sum of the numbers is larger than 6 times their difference by 6, find the two numbers.

Sol. Let the numbers be x and y , such that $x > y$.

$$\text{Then, } 3x - 4y = 5 \quad \dots(i) \text{ and } (x + y) - 6(x - y) = 6 \Rightarrow -5x + 7y = 6 \quad \dots(ii)$$

Solving (i) and (ii), we get : $x = 59$ and $y = 43$.

Hence, the required numbers are 59 and 43.

Ex. 11. The ratio between a two-digit number and the sum of the digits of that number is 4 : 1. If the digit in the unit's place is 3 more than the digit in the ten's place, what is the number ?

Sol. Let the ten's digit be x . Then, unit's digit = $(x + 3)$.

$$\text{Sum of the digits} = x + (x + 3) = 2x + 3. \text{ Number} = 10x + (x + 3) = 11x + 3.$$

$$\therefore \frac{11x + 3}{2x + 3} = \frac{4}{1} \Leftrightarrow 11x + 3 = 4(2x + 3) \Leftrightarrow 3x = 9 \Leftrightarrow x = 3.$$

$$\text{Hence, required number} = 11x + 3 = 36.$$

Ex. 12. A number consists of two digits. The sum of the digits is 9. If 63 is subtracted from the number, its digits are interchanged. Find the number.

Sol. Let the ten's digit be x . Then, unit's digit = $(9 - x)$.

$$\text{Number} = 10x + (9 - x) = 9x + 9.$$

$$\text{Number obtained by reversing the digits} = 10(9 - x) + x = 90 - 9x.$$

$$\therefore (9x + 9) - 63 = 90 - 9x \Leftrightarrow 18x = 144 \Leftrightarrow x = 8.$$

So, ten's digit = 8 and unit's digit = 1.

Hence, the required number is 81.

Ex. 13. A fraction becomes $\frac{2}{3}$ when 1 is added to both, its numerator and denominator. And, it becomes $\frac{1}{2}$ when 1 is subtracted from both the numerator and denominator. Find the fraction.

Sol. Let the required fraction be $\frac{x}{y}$. Then,

$$\frac{x+1}{y+1} = \frac{2}{3} \Rightarrow 3x - 2y = -1 \quad \dots(i) \text{ and } \frac{x-1}{y-1} = \frac{1}{2} \Rightarrow 2x - y = 1 \quad \dots(ii)$$

Solving (i) and (ii), we get : $x=3, y=5$.

$$\therefore \text{ Required fraction} = \frac{3}{5}$$

Ex. 14. 50 is divided into two parts such that the sum of their reciprocals is $\frac{1}{12}$. Find the two parts.

Sol. Let the two parts be x and $(50 - x)$.

$$\begin{aligned} \text{Then, } \frac{1}{x} + \frac{1}{50-x} &= \frac{1}{12} \Leftrightarrow \frac{50-x+x}{x(50-x)} = \frac{1}{12} \Rightarrow x^2 - 50x + 600 = 0 \\ &\Rightarrow (x-30)(x-20) = 0 \Rightarrow x = 30 \text{ or } x = 20. \end{aligned}$$

So, the parts are 30 and 20.

Ex. 15. If three numbers are added in pairs, the sums equal 10, 19 and 21. Find the numbers. (S.S.C. 2000)

Sol. Let the numbers be x, y and z . Then,

$$x + y = 10 \quad \dots(i) \quad y + z = 19 \quad \dots(ii) \quad x + z = 21 \quad \dots(iii)$$

Adding (i), (ii) and (iii), we get : $2(x + y + z) = 50$ or $(x + y + z) = 25$.

Thus, $x = (25 - 19) = 6$; $y = (25 - 21) = 4$; $z = (25 - 10) = 15$.

Hence, the required numbers are 6, 4 and 15.

8. PROBLEMS ON AGES

SOLVED EXAMPLES

Ex. 1. *Rajeev's age after 15 years will be 5 times his age 5 years back. What is the present age of Rajeev ?* (Hotel Management, 2002)

Sol. Let Rajeev's present age be x years. Then,

Rajeev's age after 15 years = $(x + 15)$ years.

Rajeev's age 5 years back = $(x - 5)$ years.

$$\therefore x + 15 = 5(x - 5) \Leftrightarrow x + 15 = 5x - 25 \Leftrightarrow 4x = 40 \Leftrightarrow x = 10.$$

Hence, Rajeev's present age = 10 years.

Ex. 2. *The ages of two persons differ by 16 years. If 6 years ago, the elder one be 3 times as old as the younger one, find their present ages.* (A.A.O. Exam, 2003)

Sol. Let the age of the younger person be x years.

Then, age of the elder person = $(x + 16)$ years.

$$\therefore 3(x - 6) = (x + 16 - 6) \Leftrightarrow 3x - 18 = x + 10 \Leftrightarrow 2x = 28 \Leftrightarrow x = 14.$$

Hence, their present ages are 14 years and 30 years.

Ex. 3. *The product of the ages of Ankit and Nikita is 240. If twice the age of Nikita is more than Ankit's age by 4 years, what is Nikita's age ?* (S.B.I.P.O. 1999)

Sol. Let Ankit's age be x years. Then, Nikita's age = $\frac{240}{x}$ years.

$$\therefore 2 \times \frac{240}{x} - x = 4 \Leftrightarrow 480 - x^2 = 4x \Leftrightarrow x^2 + 4x - 480 = 0$$

$$\Leftrightarrow (x + 24)(x - 20) = 0 \Leftrightarrow x = 20.$$

Hence, Nikita's age = $\left(\frac{240}{20}\right)$ years = 12 years.

Ex. 4. *The present age of a father is 3 years more than three times the age of his son. Three years hence, father's age will be 10 years more than twice the age of the son. Find the present age of the father.* (S.S.C. 2003)

Sol. Let the son's present age be x years. Then, father's present age = $(3x + 3)$ years.

$$\therefore (3x + 3 + 3) = 2(x + 3) + 10 \Leftrightarrow 3x + 6 = 2x + 16 \Leftrightarrow x = 10.$$

Hence, father's present age = $(3x + 3) = (3 \times 10 + 3)$ years = 33 years.

Ex. 5. *Rohit was 4 times as old as his son 8 years ago. After 8 years, Rohit will be twice as old as his son. What are their present ages ?*

Sol. Let son's age 8 years ago be x years. Then, Rohit's age 8 years ago = $4x$ years.

Son's age after 8 years = $(x + 8) + 8 = (x + 16)$ years.

Rohit's age after 8 years = $(4x + 8) + 8 = (4x + 16)$ years.

$$\therefore 2(x + 16) = 4x + 16 \Leftrightarrow 2x = 16 \Leftrightarrow x = 8.$$

Hence, son's present age = $(x + 8) = 16$ years.

Rohit's present age = $(4x + 8) = 40$ years.

Ex. 6. *One year ago, the ratio of Gaurav's and Sachin's age was 6 : 7 respectively. Four years hence, this ratio would become 7 : 8. How old is Sachin ?*

(NABARD, 2002)

Sol. Let Gaurav's and Sachin's ages one year ago be $6x$ and $7x$ years respectively. Then,
Gaurav's age 4 years hence $= (6x + 1) + 4 = (6x + 5)$ years.

Sachin's age 4 years hence $= (7x + 1) + 4 = (7x + 5)$ years.

$$\therefore \frac{6x + 5}{7x + 5} = \frac{7}{8} \Leftrightarrow 8(6x + 5) = 7(7x + 5) \Leftrightarrow 48x + 40 = 49x + 35 \Leftrightarrow x = 5$$

Hence, Sachin's present age $= (7x + 1) = 36$ years.

Ex. 7. *Abhay's age after six years will be three-seventh of his father's age. Ten years ago, the ratio of their ages was 1 : 5. What is Abhay's father's age at present ?*

Sol. Let the ages of Abhay and his father 10 years ago be x and $5x$ years respectively. Then,

Abhay's age after 6 years $= (x + 10) + 6 = (x + 16)$ years.

Father's age after 6 years $= (5x + 10) + 6 = (5x + 16)$ years.

$$\therefore \frac{(x + 16)}{(5x + 16)} = \frac{3}{7} \Leftrightarrow 7(x + 16) = 3(5x + 16) \Leftrightarrow 7x + 112 = 15x + 48$$

$$\Leftrightarrow 8x = 64 \Leftrightarrow x = 8$$

Hence, Abhay's father's present age $= (5x + 10) = 50$ years.

9. SURDS AND INDICES

IMPORTANT FACTS AND FORMULAE

1. LAWS OF INDICES :

$$(i) a^m \times a^n = a^{m+n}$$

$$(ii) \frac{a^m}{a^n} = a^{m-n}$$

$$(iii) (a^m)^n = a^{mn}$$

$$(iv) (ab)^n = a^n b^n$$

$$(v) \left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$(vi) a^0 = 1$$

2. **SURDS :** Let a be a rational number and n be a positive integer such that $a^{\frac{1}{n}} = \sqrt[n]{a}$ is irrational. Then, $\sqrt[n]{a}$ is called a surd of order n .

3. LAWS OF SURDS :

$$(i) \sqrt[n]{a} = a^{\frac{1}{n}}$$

$$(ii) \sqrt[n]{ab} = \sqrt[n]{a} \times \sqrt[n]{b}$$

$$(iii) \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$$

$$(iv) (\sqrt[n]{a})^m = a^{\frac{m}{n}}$$

$$(v) \sqrt[m]{\sqrt[n]{a}} = \sqrt[mn]{a}$$

$$(vi) (\sqrt[n]{a})^m = \sqrt[n]{a^m}$$

SOLVED EXAMPLES

Ex. 1. Simplify : (i) $(27)^{\frac{2}{3}}$ (ii) $(1024)^{-\frac{4}{5}}$ (iii) $\left(\frac{8}{125}\right)^{-\frac{4}{3}}$

Sol. (i) $(27)^{\frac{2}{3}} = (3^3)^{\frac{2}{3}} = 3^{(3 \times \frac{2}{3})} = 3^2 = 9$.

$$(ii) (1024)^{-\frac{4}{5}} = (4^5)^{-\frac{4}{5}} = 4^{\left\{5 \times \left(-\frac{4}{5}\right)\right\}} = 4^{-4} = \frac{1}{4^4} = \frac{1}{256}$$

$$(iii) \left(\frac{8}{125}\right)^{-\frac{4}{3}} = \left\{\left(\frac{2}{5}\right)^3\right\}^{-\frac{4}{3}} = \left(\frac{2}{5}\right)^{\left\{3 \times \left(-\frac{4}{3}\right)\right\}} = \left(\frac{2}{5}\right)^{-4} = \left(\frac{5}{2}\right)^4 = \frac{5^4}{2^4} = \frac{625}{16}$$

Ex. 2. Evaluate : (i) $(0.00032)^{\frac{3}{5}}$ (ii) $(256)^{0.16} \times (16)^{0.18}$

Sol. (i) $(0.00032)^{\frac{3}{5}} = \left(\frac{32}{100000}\right)^{\frac{3}{5}} = \left(\frac{2^5}{10^5}\right)^{\frac{3}{5}} = \left\{\left(\frac{2}{10}\right)^5\right\}^{\frac{3}{5}} = \left(\frac{1}{5}\right)^{(5 \times \frac{3}{5})} = \left(\frac{1}{5}\right)^3 = \frac{1}{125}$

$$(ii) (256)^{0.16} \times (16)^{0.18} = (16^2)^{0.16} \times (16)^{0.18} = (16)^{(2 \times 0.16)} \times (16)^{0.18} \\ = (16)^{0.32} \times (16)^{0.18} = (16)^{(0.32 + 0.18)} = (16)^{0.5} = (16)^{\frac{1}{2}} = 4$$

Ex. 3. What is the quotient when $(x^{-1} - 1)$ is divided by $(x - 1)$?

Sol. $\frac{x^{-1} - 1}{x - 1} = \frac{\frac{1}{x} - 1}{x - 1} = \frac{(1 - x)}{x} \times \frac{1}{(x - 1)} = -\frac{1}{x}$.

Hence, the required quotient is $-\frac{1}{x}$.

Ex. 4. If $2^{x-1} + 2^{x+1} = 1280$, then find the value of x .

Sol. $2^{x-1} + 2^{x+1} = 1280 \Leftrightarrow 2^{x-1}(1 + 2^2) = 1280$
 $\Leftrightarrow 2^{x-1} = \frac{1280}{5} = 256 = 2^8 \Leftrightarrow x - 1 = 8 \Leftrightarrow x = 9$.

Hence, $x = 9$.

Ex. 5. Find the value of $\left[5 \left(8^{\frac{1}{3}} + 27^{\frac{1}{3}}\right)^3\right]^{\frac{1}{4}}$.

Sol. $\left[5 \left(8^{\frac{1}{3}} + 27^{\frac{1}{3}}\right)^3\right]^{\frac{1}{4}} = \left[5 \left\{(2^3)^{\frac{1}{3}} + (3^3)^{\frac{1}{3}}\right\}^3\right]^{\frac{1}{4}} = \left[5 \left\{2^{\left(3 \times \frac{1}{3}\right)} + 3^{\left(3 \times \frac{1}{3}\right)}\right\}^3\right]^{\frac{1}{4}}$
 $= \left[5 (2 + 3)^3\right]^{\frac{1}{4}} = (5 \times 5^3)^{\frac{1}{4}} = (5^4)^{\frac{1}{4}} = 5^{\left(4 \times \frac{1}{4}\right)} = 5^1 = 5$.

Ex. 6. Find the value of $\left\{(16)^{\frac{3}{2}} + (16)^{-\frac{3}{2}}\right\}$.

Sol. $\left[(16)^{\frac{3}{2}} + (16)^{-\frac{3}{2}}\right] = \left[(4^2)^{\frac{3}{2}} + (4^2)^{-\frac{3}{2}}\right] = 4^{\left(2 \times \frac{3}{2}\right)} + 4^{\left\{2 \times \left(-\frac{3}{2}\right)\right\}}$
 $= 4^3 + 4^{-3} = 4^3 + \frac{1}{4^3} = \left(64 + \frac{1}{64}\right) = \frac{4097}{64}$.

Ex. 7. If $\left(\frac{1}{5}\right)^{3y} = 0.008$, then find the value of $(0.25)^y$.

Sol. $\left(\frac{1}{5}\right)^{3y} = 0.008 = \frac{8}{1000} = \frac{1}{125} = \left(\frac{1}{5}\right)^3 \Leftrightarrow 3y = 3 \Leftrightarrow y = 1$
 $\therefore (0.25)^y = (0.25)^1 = 0.25$.

Ex. 8. Find the value of $\frac{(243)^{\frac{n}{5}} \cdot 3^{2n+1}}{9^n \times 3^{n-1}}$.

Sol. $\frac{(243)^{\frac{n}{5}} \cdot 3^{2n+1}}{9^n \times 3^{n-1}} = \frac{(3^5)^{\frac{n}{5}} \times 3^{2n+1}}{(3^2)^n \times 3^{n-1}} = \frac{3^{\left(5 \times \frac{n}{5}\right)} \times 3^{2n+1}}{3^{2n} \times 3^{n-1}} = \frac{3^n \times 3^{2n+1}}{3^{2n} \times 3^{n-1}}$
 $= \frac{3^{n+(2n+1)}}{3^{2n+n-1}} = \frac{3^{(3n+1)}}{3^{(3n-1)}} = 3^{(3n+1)-(3n-1)} = 3^2 = 9$.

Ex. 9. Find the value of $\left(2^4 - 1\right)\left(2^4 + 2^2 + 2^4 + 1\right)$.

(N.I.F.T. 2003)

Sol. Putting $2^{\frac{1}{4}} = x$, we get :

$$\begin{aligned} \left(\frac{1}{2^4} - 1 \right) \left(\frac{3}{2^4} + \frac{1}{2^2} + \frac{1}{2^4} + 1 \right) &= (x-1)(x^3+x^2+x+1), \text{ where } x = 2^{\frac{1}{4}} \\ &= (x-1)[x^2(x+1)+(x+1)] \\ &= (x-1)(x+1)(x^2+1) = (x^2-1)(x^2+1) \\ &= (x^4-1) = \left[\left(\frac{1}{2^4} \right)^4 - 1 \right] = \left[2^{\left(\frac{1}{4} \times 4 \right)} - 1 \right] = (2-1) = 1. \end{aligned}$$

Ex. 10. Find the value of $\frac{6^{\frac{2}{3}} \times \sqrt[3]{6^7}}{\sqrt[3]{6^6}}$.

$$\begin{aligned} \text{Sol. } \frac{6^{\frac{2}{3}} \times \sqrt[3]{6^7}}{\sqrt[3]{6^6}} &= \frac{6^{\frac{2}{3}} \times (6^7)^{\frac{1}{3}}}{(6^6)^{\frac{1}{3}}} = \frac{6^{\frac{2}{3}} \times 6^{\left(\frac{7 \times 1}{3} \right)}}{6^{\left(6 \times \frac{1}{3} \right)}} = \frac{6^{\frac{2}{3}} \times 6^{\left(\frac{7}{3} \right)}}{6^2} \\ &= 6^{\frac{2}{3}} \times 6^{\left(\frac{7}{3} - 2 \right)} = 6^{\frac{2}{3}} \times 6^{\frac{1}{3}} = 6^{\left(\frac{2}{3} + \frac{1}{3} \right)} = 6^1 = 6. \end{aligned}$$

10. PERCENTAGE

IMPORTANT FACTS AND FORMULAE

- I. **Concept of Percentage** : By a certain *percent*, we mean that many hundredths. Thus, x percent means x hundredths, written as $x\%$.

To express $x\%$ as a fraction : We have, $x\% = \frac{x}{100}$.

Thus, $20\% = \frac{20}{100} = \frac{1}{5}$; $48\% = \frac{48}{100} = \frac{12}{25}$, etc.

To express $\frac{a}{b}$ as a percent : We have, $\frac{a}{b} = \left(\frac{a}{b} \times 100\right)\%$.

Thus, $\frac{1}{4} = \left(\frac{1}{4} \times 100\right)\% = 25\%$; $0.6 = \frac{6}{10} = \frac{3}{5} = \left(\frac{3}{5} \times 100\right)\% = 60\%$.

- II. If the price of a commodity increases by $R\%$, then the reduction in consumption so as not to increase the expenditure is

$$\left[\frac{R}{(100 + R)} \times 100 \right] \%$$

If the price of a commodity decreases by $R\%$, then the increase in consumption so as not to decrease the expenditure is

$$\left[\frac{R}{(100 - R)} \times 100 \right] \%$$

- III. **Results on Population** : Let the population of a town be P now and suppose it increases at the rate of $R\%$ per annum, then :

1. Population after n years $= P \left(1 + \frac{R}{100}\right)^n$.

2. Population n years ago $= \frac{P}{\left(1 + \frac{R}{100}\right)^n}$.

- IV. **Results on Depreciation** : Let the present value of a machine be P . Suppose it depreciates at the rate of $R\%$ per annum. Then :

1. Value of the machine after n years $= P \left(1 - \frac{R}{100}\right)^n$.

2. Value of the machine n years ago $= \frac{P}{\left(1 - \frac{R}{100}\right)^n}$.

- V. If A is $R\%$ more than B , then B is less than A by

$$\left[\frac{R}{(100 + R)} \times 100 \right] \%$$

If A is $R\%$ less than B , then B is more than A by

$$\left[\frac{R}{(100 - R)} \times 100 \right] \%$$

SOLVED EXAMPLES

Ex. 1. Express each of the following as a fraction :

- (i) 56% (ii) 4% (iii) 0.6% (iv) 0.08%

Sol. (i) $56\% = \frac{56}{100} = \frac{14}{25}$ (ii) $4\% = \frac{4}{100} = \frac{1}{25}$
 (iii) $0.6\% = \frac{0.6}{100} = \frac{6}{1000} = \frac{3}{500}$ (iv) $0.08\% = \frac{0.08}{100} = \frac{8}{10000} = \frac{1}{1250}$

Ex. 2. Express each of the following as a decimal :

- (i) 6% (ii) 28% (iii) 0.2% (iv) 0.04%

Sol. (i) $6\% = \frac{6}{100} = 0.06$ (ii) $28\% = \frac{28}{100} = 0.28$
 (iii) $0.2\% = \frac{0.2}{100} = 0.002$ (iv) $0.04\% = \frac{0.04}{100} = 0.0004$

Ex. 3. Express each of the following as rate percent :

- (i) $\frac{23}{36}$ (ii) $6\frac{3}{4}$ (iii) 0.004

Sol. (i) $\frac{23}{36} = \left(\frac{23}{36} \times 100\right)\% = \left(\frac{575}{9}\right)\% = 63\frac{8}{9}\%$
 (ii) $0.004 = \frac{4}{1000} = \left(\frac{4}{1000} \times 100\right)\% = 0.4\%$
 (iii) $6\frac{3}{4} = \frac{27}{4} = \left(\frac{27}{4} \times 100\right)\% = 675\%$

Ex. 4. Evaluate :

- (i) 28% of 450 + 45% of 280

(Bank P.O. 2003)

- (ii) $16\frac{2}{3}\%$ of 600 gm - $33\frac{1}{3}\%$ of 180 gm

(R.R.B. 1998)

Sol. (i) $28\% \text{ of } 450 + 45\% \text{ of } 280 = \left(\frac{28}{100} \times 450 + \frac{45}{100} \times 280\right) = (126 + 126) = 252$

- (ii) $16\frac{2}{3}\%$ of 600 gm - $33\frac{1}{3}\%$ of 180 gm

$= \left[\left(\frac{50}{3} \times \frac{1}{100} \times 600\right) - \left(\frac{100}{3} \times \frac{1}{100} \times 180\right)\right] \text{ gm} = (100 - 60) \text{ gm} = 40 \text{ gm}$

Ex. 5. (i) 2 is what percent of 50 ?

(S.S.C. 2000)

- (ii) $\frac{1}{2}$ is what percent of $\frac{1}{3}$?

(S.S.C. 2002)

- (iii) What percent of 7 is 84 ?

- (iv) What percent of 2 metric tonnes is 40 quintals ?

- (v) What percent of 6.5 litres is 130 ml ?

Sol. (i) Required percentage = $\left(\frac{2}{50} \times 100\right)\% = 4\%$

(ii) Required percentage = $\left(\frac{1}{2} \times \frac{3}{1} \times 100\right)\% = 150\%$

(iii) Required percentage = $\left(\frac{84}{7} \times 100\right)\% = 1200\%$

(iv) 1 metric tonne = 10 quintals.

$$\therefore \text{Required percentage} = \left(\frac{40}{2 \times 10} \times 100 \right) \% = 200\%.$$

$$(v) \text{ Required percentage} = \left(\frac{130}{6.5 \times 1000} \times 100 \right) \% = 2\%.$$

Ex. 6. Find the missing figures :

(i) ?% of 25 = 2.125 (ii) 9% of ? = 63 (iii) 0.25% of ? = 0.04

Sol. (i) Let $x\%$ of 25 = 2.125. Then, $\frac{x}{100} \times 25 = 2.125 \Leftrightarrow x = (2.125 \times 4) = 8.5$.

(ii) Let 9% of $x = 6.3$. Then, $\frac{9}{100} x = 6.3 \Leftrightarrow x = \left(\frac{6.3 \times 100}{9} \right) = 70$.

(iii) Let 0.25% of $x = 0.04$. Then, $\frac{0.25}{100} x = 0.04 \Leftrightarrow x = \left(\frac{0.04 \times 100}{0.25} \right) = 16$.

Ex. 7. Which is greatest in $16\frac{2}{3}\%$, $\frac{2}{15}$ and 0.17 ?

Sol. $16\frac{2}{3}\% = \left(\frac{50}{3} \times \frac{1}{100} \right) = \frac{1}{6} = 0.166$, $\frac{2}{15} = 0.133$. Clearly, 0.17 is the greatest.

Ex. 8. If the sales tax be reduced from $3\frac{1}{2}\%$ to $3\frac{1}{3}\%$, then what difference does it make to a person who purchases an article with marked price of Rs. 8400 ?

(S.S.C. 2002)

Sol. Required difference = $\left(3\frac{1}{2}\% \text{ of Rs. 8400} \right) - \left(3\frac{1}{3}\% \text{ of Rs. 8400} \right)$
 $= \left(\frac{7}{2} - \frac{10}{3} \right) \% \text{ of Rs. 8400} = \frac{1}{6} \% \text{ of Rs. 8400}$
 $= \text{Rs. } \left(\frac{1}{6} \times \frac{1}{100} \times 8400 \right) = \text{Rs. 14}.$

Ex. 9. An inspector rejects 0.08% of the meters as defective. How many will he examine to reject 2 ?

(M.A.T. 2000)

Sol. Let the number of meters to be examined be x .

$$\text{Then, } 0.08\% \text{ of } x = 2 \Leftrightarrow \left(\frac{8}{100} \times \frac{1}{100} \times x \right) = 2 \Leftrightarrow x = \left(\frac{2 \times 100 \times 100}{8} \right) = 2500.$$

Ex. 10. Sixty-five percent of a number is 21 less than four-fifth of that number. What is the number ?

Sol. Let the number be x .

$$\text{Then, } \frac{4}{5}x - (65\% \text{ of } x) = 21 \Leftrightarrow \frac{4}{5}x - \frac{65}{100}x = 21 \Leftrightarrow 15x = 2100 \Leftrightarrow x = 140.$$

Ex. 11. Difference of two numbers is 1660. If 7.5% of one number is 12.5% of the other number, find the two numbers.

Sol. Let the numbers be x and y . Then, $7.5\% \text{ of } x = 12.5\% \text{ of } y \Leftrightarrow x = \frac{125}{75}y = \frac{5}{3}y$.

$$\text{Now, } x - y = 1660 \Rightarrow \frac{5}{3}y - y = 1660 \Rightarrow \frac{2}{3}y = 1660 \Rightarrow y = \left(\frac{1660 \times 3}{2} \right) = 2490.$$

$$\therefore \text{ One number} = 2490, \text{ Second number} = \frac{5}{3}y = 4150.$$

Ex. 12. In expressing a length 81.472 km as nearly as possible with three significant digits, find the percentage error. (S.S.C. 1997)

Sol. Error = $(81.5 - 81.472)$ km = 0.028.

$$\therefore \text{Required percentage} = \left(\frac{0.028}{81.472} \times 100 \right) \% = 0.034\%.$$

Ex. 13. In an election between two candidates, 75% of the voters cast their votes, out of which 2% of the votes were declared invalid. A candidate got 9261 votes which were 75% of the total valid votes. Find the total number of votes enrolled in that election. (S.S.C. 2003)

Sol. Let the total number of votes enrolled be x . Then,

Number of votes cast = 75% of x . Valid votes = 98% of (75% of x).

$$\therefore 75\% \text{ of } [98\% \text{ of } (75\% \text{ of } x)] = 9261$$

$$\Leftrightarrow \left(\frac{75}{100} \times \frac{98}{100} \times \frac{75}{100} \times x \right) = 9261 \Leftrightarrow x = \left(\frac{9261 \times 100 \times 100 \times 100}{75 \times 98 \times 75} \right) = 16800.$$

Ex. 14. Shobha's Mathematics Test had 75 problems i.e., 10 arithmetic, 30 algebra and 35 geometry problems. Although she answered 70% of the arithmetic, 40% of the algebra and 60% of the geometry problems correctly, she did not pass the test because she got less than 60% of the problems right. How many more questions she would have needed to answer correctly to earn a 60% passing grade? (C.D.S. 2002)

Sol. Number of questions attempted correctly = (70% of 10 + 40% of 30 + 60% of 35)
 $= (7 + 12 + 21) = 40.$

Questions to be answered correctly for 60% grade = 60% of 75 = 45.

$$\therefore \text{Required number of questions} = (45 - 40) = 5.$$

Ex. 15. If 50% of $(x - y) = 30\%$ of $(x + y)$, then what percent of x is y ? (S.S.C. 2003)

$$\begin{aligned} \text{Sol. } 50\% \text{ of } (x - y) &= 30\% \text{ of } (x + y) \Leftrightarrow \frac{50}{100} (x - y) = \frac{30}{100} (x + y) \\ \Leftrightarrow 5(x - y) &= 3(x + y) \Leftrightarrow 2x = 8y \Leftrightarrow x = 4y. \end{aligned}$$

$$\therefore \text{Required percentage} = \left(\frac{y}{x} \times 100 \right) \% = \left(\frac{y}{4y} \times 100 \right) \% = 25\%.$$

Ex. 16. Mr. Jones gave 40% of the money he had, to his wife. He also gave 20% of the remaining amount to each of his three sons. Half of the amount now left was spent on miscellaneous items and the remaining amount of Rs. 12,000 was deposited in the bank. How much money did Mr. Jones have initially?

Sol. Let the initial amount with Mr. Jones be Rs. x . Then,

$$\text{Money given to wife} = \text{Rs. } \frac{40}{100} x = \text{Rs. } \frac{2x}{5}. \text{ Balance} = \text{Rs. } \left(x - \frac{2x}{5} \right) = \text{Rs. } \frac{3x}{5}.$$

$$\text{Money given to 3 sons} = \text{Rs. } \left[3 \times \left(\frac{20}{100} \times \frac{3x}{5} \right) \right] = \text{Rs. } \frac{9x}{25}.$$

$$\text{Balance} = \text{Rs. } \left(\frac{3x}{5} - \frac{9x}{25} \right) = \text{Rs. } \frac{6x}{25}.$$

$$\text{Amount deposited in bank} = \text{Rs. } \left(\frac{1}{2} \times \frac{6x}{25} \right) = \text{Rs. } \frac{3x}{25}.$$

$$\therefore \frac{3x}{25} = 12000 \Leftrightarrow x = \left(\frac{12000 \times 25}{3} \right) = 100000.$$

So, Mr. Jones initially had Rs. 1,00,000 with him.

Short-cut Method : Let the initial amount with Mr. Jones be Rs. x .

Then, $\frac{1}{2} [100 - (3 \times 20)]\%$ of $(100 - 40)\%$ of $x = 12000$.

$$\Leftrightarrow \frac{1}{2} \times \frac{40}{100} \times \frac{60}{100} \times x = 12000 \Leftrightarrow \frac{3}{25} x = 12000 \Leftrightarrow x = \left(\frac{12000 \times 25}{3} \right) = 100000.$$

Ex. 17. 10% of the inhabitants of a village having died of cholera, a panic set in, during which 25% of the remaining inhabitants left the village. The population is then reduced to 4050. Find the number of original inhabitants. (S.S.C. 2002)

Sol. Let the total number of original inhabitants be x .

Then, $(100 - 25)\%$ of $(100 - 10)\%$ of $x = 4050$

$$\Leftrightarrow \left(\frac{75}{100} \times \frac{90}{100} \times x \right) = 4050 \Leftrightarrow \frac{27}{40} x = 4050 \Leftrightarrow x = \left(\frac{4050 \times 40}{27} \right) = 6000.$$

\therefore Number of original inhabitants = 6000.

Ex. 18. A salesman's commission is 5% on all sales upto Rs. 10,000 and 4% on all sales exceeding this. He remits Rs. 31,100 to his parent company after deducting his commission. Find the total sales. (R.R.B. 2001)

Sol. Let his total sales be Rs. x . Now, (Total Sales) - (Commission) = Rs. 31,100.

$\therefore x - [5\% \text{ of } 10000 + 4\% \text{ of } (x - 10000)] = 31100$

$$\Leftrightarrow x - \left[\frac{5}{100} \times 10000 + \frac{4}{100} (x - 10000) \right] = 31100 \Leftrightarrow x - 500 - \frac{(x - 10000)}{25} = 31100$$

$$\Leftrightarrow x - \frac{x}{25} = 31200 \Leftrightarrow \frac{24x}{25} = 31200 \Leftrightarrow x = \left(\frac{31200 \times 25}{24} \right) = 32500.$$

\therefore Total sales = Rs. 32,500.

Ex. 19. Raman's salary was decreased by 50% and subsequently increased by 50%. How much percent does he lose? (Hotel Management, 2003)

Sol. Let original salary = Rs. 100.

$$\text{New final salary} = 150\% \text{ of } (50\% \text{ of Rs. } 100) = \text{Rs. } \left(\frac{150}{100} \times \frac{50}{100} \times 100 \right) = \text{Rs. } 75.$$

\therefore Decrease = 25%.

Ex. 20. Paulson spends 75% of his income. His income is increased by 20% and he increased his expenditure by 10%. Find the percentage increase in his savings.

Sol. Let original income = Rs. 100. Then, expenditure = Rs. 75 and savings = Rs. 25.

$$\text{New income} = \text{Rs. } 120, \text{ New expenditure} = \text{Rs. } \left(\frac{110}{100} \times 75 \right) = \text{Rs. } \frac{165}{2}.$$

$$\text{New savings} = \text{Rs. } \left(120 - \frac{165}{2} \right) = \text{Rs. } \frac{75}{2}.$$

$$\text{Increase in savings} = \text{Rs. } \left(\frac{75}{2} - 25 \right) = \text{Rs. } \frac{25}{2}.$$

$$\therefore \text{Increase\%} = \left(\frac{25}{2} \times \frac{1}{25} \times 100 \right)\% = 50\%.$$

Ex. 21. The salary of a person was reduced by 10%. By what percent should his reduced salary be raised so as to bring it at par with his original salary? (S.S.C. 2004)

Sol. Let the original salary be Rs. 100. New salary = Rs. 90.

$$\text{Increase on } 90 = 10. \text{ Increase on } 100 = \left(\frac{10}{90} \times 100 \right)\% = 11\frac{1}{9}\%.$$

11. PROFIT AND LOSS

IMPORTANT FACTS

Cost Price : The price at which an article is purchased, is called its **cost price**, abbreviated as **C.P.**

Selling Price : The price at which an article is sold, is called its **selling price**, abbreviated as **S.P.**

Profit or Gain : If S.P. is greater than C.P., the seller is said to have a **profit** or **gain**.

Loss : If S.P. is less than C.P., the seller is said to have incurred a **loss**.

FORMULAE

1. $\text{Gain} = (\text{S.P.}) - (\text{C.P.})$
2. $\text{Loss} = (\text{C.P.}) - (\text{S.P.})$
3. Loss or gain is always reckoned on C.P.
4. $\text{Gain}\% = \left(\frac{\text{Gain} \times 100}{\text{C.P.}} \right)$
5. $\text{Loss}\% = \left(\frac{\text{Loss} \times 100}{\text{C.P.}} \right)$
6. $\text{S.P.} = \frac{(100 + \text{Gain}\%)}{100} \times \text{C.P.}$
7. $\text{S.P.} = \frac{(100 - \text{Loss}\%)}{100} \times \text{C.P.}$
8. $\text{C.P.} = \frac{100}{(100 + \text{Gain}\%)} \times \text{S.P.}$
9. $\text{C.P.} = \frac{100}{(100 - \text{Loss}\%)} \times \text{S.P.}$
10. If an article is sold at a gain of say, 35%, then S.P. = 135% of C.P.
11. If an article is sold at a loss of say, 35%, then S.P. = 65% of C.P.
12. When a person sells two similar items, one at a gain of say, $x\%$, and the other at a loss of $x\%$, then the seller always incurs a loss given by :

$$\text{Loss}\% = \left(\frac{\text{Common Loss and Gain}\%}{10} \right)^2 = \left(\frac{x}{10} \right)^2$$

13. If a trader professes to sell his goods at cost price, but uses false weights, then

$$\text{Gain}\% = \left[\frac{\text{Error}}{(\text{True Value}) - (\text{Error})} \times 100 \right] \%$$

SOLVED EXAMPLES

Ex. 1. A man buys an article for Rs. 27.50 and sells it for Rs. 28.60. Find his gain percent.

Sol. C.P. = Rs. 27.50, S.P. = Rs. 28.60.

So, Gain = Rs. (28.60 - 27.50) = Rs. 1.10.

$$\therefore \text{Gain}\% = \left(\frac{1.10}{27.50} \times 100 \right) \% = 4\%$$

Ex. 2. If a radio is purchased for Rs. 490 and sold for Rs. 465.50, find the loss percent.

Sol. C.P. = Rs. 490, S.P. = Rs. 465.50.

Loss = Rs. (490 - 465.50) = Rs. 24.50.

$$\therefore \text{Loss \%} = \left(\frac{24.50}{490} \times 100 \right) \% = 5\%.$$

Ex. 3. Find S.P., when

(i) C.P. = Rs. 56.25, Gain = 20%

(ii) C.P. = Rs. 80.40, Loss = 5%

Sol. (i) S.P. = 120% of Rs. 56.25 = Rs. $\left(\frac{120}{100} \times 56.25 \right)$ = Rs. 67.50.

(ii) S.P. = 85% of Rs. 80.40 = Rs. $\left(\frac{85}{100} \times 80.40 \right)$ = Rs. 68.34.

Ex. 4. Find C.P., when

(i) S.P. = Rs. 40.60, Gain = 16%

(ii) S.P. = Rs. 51.70, Loss = 12%

Sol. (i) C.P. = Rs. $\left(\frac{100}{116} \times 40.60 \right)$ = Rs. 35.

(ii) C.P. = Rs. $\left(\frac{100}{88} \times 51.70 \right)$ = Rs. 58.75.

Ex. 5. A person incurs 5% loss by selling a watch for Rs. 1140. At what price should the watch be sold to earn 5% profit? (R.R.B. 2002)

Sol. Let the new S.P. be Rs. x . Then,

$$(100 - \text{loss\%}) : (\text{1st S.P.}) = (100 + \text{gain\%}) : (\text{2nd S.P.})$$

$$\Rightarrow \left(\frac{100 - 5}{1140} \right) = \left(\frac{100 + 5}{x} \right) \Rightarrow x = \left(\frac{105 \times 1140}{95} \right) = 1260.$$

\therefore New S.P. = Rs. 1260.

Ex. 6. A book was sold for Rs. 27.50 with a profit of 10%. If it were sold for Rs. 25.75, then what would have been the percentage of profit or loss? (Hotel Management, 2003)

Sol. S.P. = Rs. 27.50, Profit = 10%.

So, C.P. = Rs. $\left(\frac{100}{110} \times 27.50 \right)$ = Rs. 25.

When S.P. = Rs. 25.75, profit = Rs. (25.75 - 25) = Re. 0.75.

$$\therefore \text{Profit \%} = \left(\frac{0.75}{25} \times 100 \right) \% = 3\%.$$

Ex. 7. If the cost price is 96% of the selling price, then what is the profit percent?

Sol. Let S.P. = Rs. 100. Then, C.P. = Rs. 96; Profit = Rs. 4.

$$\therefore \text{Profit \%} = \left(\frac{4}{96} \times 100 \right) \% = \frac{25}{6} \% = 4.17\%.$$

Ex. 8. The C.P. of 21 articles is equal to S.P. of 18 articles. Find the gain or loss percent.

Sol. Let C.P. of each article be Re. 1.

Then, C.P. of 18 articles = Rs. 18, S.P. of 18 articles = Rs. 21.

$$\therefore \text{Gain \%} = \left(\frac{3}{18} \times 100 \right) \% = 16\frac{2}{3}\%.$$

Ex. 9. By selling 33 metres of cloth, one gains the selling price of 11 metres. Find the gain percent. (Section Officers', 2001)

Sol. (S.P. of 33 m) - (C.P. of 33 m) = Gain = S.P. of 11 m.

\therefore S.P. of 22 m = C.P. of 33 m.

Let C.P. of each metre be Re. 1. Then, C.P. of 22 m = Rs. 22, S.P. of 22 m = Rs. 33.

\therefore Gain% = $\left(\frac{11}{22} \times 100\right)\% = 50\%$.

Ex. 10. A vendor bought bananas at 6 for Rs. 10 and sold them at 4 for Rs. 6. Find his gain or loss percent.

Sol. Suppose, number of bananas bought = L.C.M. of 6 and 4 = 12.

\therefore C.P. = Rs. $\left(\frac{10}{6} \times 12\right)$ = Rs. 20; S.P. = Rs. $\left(\frac{6}{4} \times 12\right)$ = Rs. 18.

\therefore Loss% = $\left(\frac{2}{20} \times 100\right)\% = 10\%$.

Ex. 11. A man bought toffees at 3 for a rupee. How many for a rupee must he sell to gain 50%?

Sol. C.P. of 3 toffees = Re. 1; S.P. of 3 toffees = 150% of Re. 1 = $\frac{3}{2}$.

For Rs. $\frac{3}{2}$, toffees sold = 3. For Re. 1, toffees sold = $\left(3 \times \frac{2}{3}\right) = 2$.

Ex. 12. A grocer purchased 80 kg of sugar at Rs. 13.50 per kg and mixed it with 120 kg sugar at Rs. 16 per kg. At what rate should he sell the mixture to gain 16%?

Sol. C.P. of 200 kg of mixture = Rs. $(80 \times 13.50 + 120 \times 16)$ = Rs. 3000.

S.P. = 116% of Rs. 3000 = Rs. $\left(\frac{116}{100} \times 3000\right)$ = Rs. 3480.

\therefore Rate of S.P. of the mixture = Rs. $\left(\frac{3480}{200}\right)$ per kg = Rs. 17.40 per kg.

Ex. 13. Pure ghee costs Rs. 100 per kg. After adulterating it with vegetable oil costing Rs. 50 per kg, a shopkeeper sells the mixture at the rate of Rs. 96 per kg, thereby making a profit of 20%. In what ratio does he mix the two?

Sol. Mean cost price = Rs. $\left(\frac{100}{120} \times 96\right)$ = Rs. 80 per kg.

By the rule of alligation :

C.P. of 1 kg ghee

C.P. of 1 kg oil

100

50

Mean price

80

30

20

\therefore Required ratio = 30 : 20 = 3 : 2.

Ex. 14. A dishonest dealer professes to sell his goods at cost price but uses a weight of 960 gms for a kg. weight. Find his gain percent.

Sol. Gain% = $\left[\frac{\text{Error}}{(\text{True Value}) - (\text{Error})} \times 100\right]\% = \left(\frac{40}{960} \times 100\right)\% = 4\frac{1}{6}\%$.

12. RATIO AND PROPORTION

IMPORTANT FACTS AND FORMULAE

- I. **RATIO** : The ratio of two quantities a and b in the same units, is the fraction $\frac{a}{b}$ and we write it as $a : b$.
In the ratio $a : b$, we call a as the **first term** or **antecedent** and b , the **second term** or **consequent**.
Ex. The ratio $5 : 9$ represents $\frac{5}{9}$ with antecedent = 5, consequent = 9.
Rule : The multiplication or division of each term of a ratio by the same non-zero number does not affect the ratio.
Ex. $4 : 5 = 8 : 10 = 12 : 15$ etc. Also, $4 : 6 = 2 : 3$.
2. **PROPORTION** : The equality of two ratios is called **proportion**.
If $a : b = c : d$, we write, $a : b :: c : d$ and we say that a, b, c, d are in proportion. Here a and d are called **extremes**, while b and c are called **mean terms**.
Product of means = Product of extremes.
Thus, $a : b :: c : d \Leftrightarrow (b \times c) = (a \times d)$.
3. (i) **Fourth Proportional** : If $a : b = c : d$, then d is called the fourth proportional to a, b, c .
(ii) **Third Proportional** : If $a : b = b : c$, then c is called the third proportional to a and b .
(iii) **Mean Proportional** : Mean proportional between a and b is \sqrt{ab} .
4. (i) **COMPARISON OF RATIOS** :
We say that $(a : b) > (c : d) \Leftrightarrow \frac{a}{b} > \frac{c}{d}$.
(ii) **COMPOUNDED RATIO** :
The compounded ratio of the ratios $(a : b), (c : d), (e : f)$ is $(ace : bdf)$.
5. (i) **Duplicate ratio** of $(a : b)$ is $(a^2 : b^2)$.
(ii) **Sub-duplicate ratio** of $(a : b)$ is $(\sqrt{a} : \sqrt{b})$.
(iii) **Triplicate ratio** of $(a : b)$ is $(a^3 : b^3)$.
(iv) **Sub-triplicate ratio** of $(a : b)$ is $\left(\frac{1}{a^3} : \frac{1}{b^3}\right)$.
(v) If $\frac{a}{b} = \frac{c}{d}$, then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$. (**componendo and dividendo**)
6. **VARIATION** :
(i) We say that x is directly proportional to y , if $x = ky$ for some constant k and we write, $x \propto y$.
(ii) We say that x is inversely proportional to y , if $xy = k$ for some constant k and we write, $x \propto \frac{1}{y}$.

SOLVED PROBLEMS

Ex. 1. If $a : b = 5 : 9$ and $b : c = 4 : 7$, find $a : b : c$.

Sol. $a : b = 5 : 9$ and $b : c = 4 : 7 = \left(4 \times \frac{9}{4}\right) : \left(7 \times \frac{9}{4}\right) = 9 : \frac{63}{4}$
 $\Rightarrow a : b : c = 5 : 9 : \frac{63}{4} = 20 : 36 : 63.$

Ex. 2. Find :

- (i) the fourth proportional to 4, 9, 12;
- (ii) the third proportional to 16 and 36;
- (iii) the mean proportional between 0.08 and 0.18.

Sol. (i) Let the fourth proportional to 4, 9, 12 be x .

Then, $4 : 9 :: 12 : x \Leftrightarrow 4 \times x = 9 \times 12 \Leftrightarrow x = \frac{9 \times 12}{4} = 27.$

\therefore Fourth proportional to 4, 9, 12 is 27.

(ii) Let the third proportional to 16 and 36 be x .

Then, $16 : 36 :: 36 : x \Leftrightarrow 16 \times x = 36 \times 36 \Leftrightarrow x = \frac{36 \times 36}{16} = 81.$

\therefore Third proportional to 16 and 36 is 81.

(iii) Mean proportional between 0.08 and 0.18

$= \sqrt{0.08 \times 0.18} = \sqrt{\frac{8}{100} \times \frac{18}{100}} = \sqrt{\frac{144}{100 \times 100}} = \frac{12}{100} = 0.12.$

Ex. 3. If $x : y = 3 : 4$, find $(4x + 5y) : (5x - 2y)$.

Sol. $\frac{x}{y} = \frac{3}{4} \Rightarrow \frac{4x + 5y}{5x - 2y} = \frac{4\left(\frac{x}{y}\right) + 5}{5\left(\frac{x}{y}\right) - 2} = \frac{4 \times \frac{3}{4} + 5}{5 \times \frac{3}{4} - 2} = \frac{(3 + 5)}{\left(\frac{7}{4}\right)} = \frac{32}{7}.$

Ex. 4. Divide Rs. 672 in the ratio 5 : 3.

Sol. Sum of ratio terms = $(5 + 3) = 8.$

\therefore First part = Rs. $\left(672 \times \frac{5}{8}\right)$ = Rs. 420; Second part = Rs. $\left(672 \times \frac{3}{8}\right)$ = Rs. 252.

Ex. 5. Divide Rs. 1162 among A, B, C in the ratio 35 : 28 : 20.

Sol. Sum of ratio terms = $(35 + 28 + 20) = 83.$

A's share = Rs. $\left(1162 \times \frac{35}{83}\right)$ = Rs. 490; B's share = Rs. $\left(1162 \times \frac{28}{83}\right)$ = Rs. 392;

C's share = Rs. $\left(1162 \times \frac{20}{83}\right)$ = Rs. 280.

Ex. 6. A bag contains 50 p, 25 p and 10 p coins in the ratio 5 : 9 : 4, amounting to Rs. 206. Find the number of coins of each type.

Sol. Let the number of 50 p, 25 p and 10 p coins be $5x$, $9x$ and $4x$ respectively.

Then, $\frac{5x}{2} + \frac{9x}{4} + \frac{4x}{10} = 206$

$\Leftrightarrow 50x + 45x + 8x = 4120 \Leftrightarrow 103x = 4120 \Leftrightarrow x = 40.$

\therefore Number of 50 p coins = $(5 \times 40) = 200$; Number of 25 p coins = $(9 \times 40) = 360$;

Number of 10 p coins = $(4 \times 40) = 160.$

Ex. 7. A mixture contains alcohol and water in the ratio 4 : 3. If 5 litres of water is added to the mixture, the ratio becomes 4 : 5. Find the quantity of alcohol in the given mixture.

Sol. Let the quantity of alcohol and water be $4x$ litres and $3x$ litres respectively. Then,

$$\frac{4x}{3x+5} = \frac{4}{5} \Leftrightarrow 20x = 4(3x+5) \Leftrightarrow 8x = 20 \Leftrightarrow x = 2.5.$$

\therefore Quantity of alcohol = (4×2.5) litres = 10 litres.

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13. PARTNERSHIP

IMPORTANT FACTS AND FORMULAE

- Partnership** : When two or more than two persons run a business jointly, they are called **partners** and the deal is known as **partnership**.
- Ratio of Division of Gains** :
 - When investments of all the partners are for the same time, the gain or loss is distributed among the partners in the ratio of their investments.
 Suppose A and B invest Rs. x and Rs. y respectively for a year in a business, then at the end of the year :
 (A's share of profit) : (B's share of profit) = $x : y$.
 - When investments are for different time periods, then equivalent capitals are calculated for a unit of time by taking (capital \times number of units of time). Now, gain or loss is divided in the ratio of these capitals.
 Suppose A invests Rs. x for p months and B invests Rs. y for q months, then
 (A's share of profit) : (B's share of profit) = $xp : yq$.
- Working and Sleeping Partners** : A partner who manages the business is known as a **working partner** and the one who simply invests the money is a **sleeping partner**.

SOLVED EXAMPLES

Ex. 1. A, B and C started a business by investing Rs. 1,20,000, Rs. 1,35,000 and Rs. 1,50,000 respectively. Find the share of each, out of an annual profit of Rs. 56,700.

Sol. Ratio of shares of A, B and C = Ratio of their investments
 $= 120000 : 135000 : 150000 = 8 : 9 : 10$.

$$\therefore \text{A's share} = \text{Rs.} \left(56700 \times \frac{8}{27} \right) = \text{Rs.} 16800.$$

$$\text{B's share} = \text{Rs.} \left(56700 \times \frac{9}{27} \right) = \text{Rs.} 18900.$$

$$\text{C's share} = \text{Rs.} \left(56700 \times \frac{10}{27} \right) = \text{Rs.} 21000.$$

Ex. 2. Alfred started a business investing Rs. 45,000. After 3 months, Peter joined him with a capital of Rs. 60,000. After another 6 months, Ronald joined them with a capital of Rs. 90,000. At the end of the year, they made a profit of Rs. 16,500. Find the share of each.

Sol. Clearly, Alfred invested his capital for 12 months, Peter for 9 months and Ronald for 3 months.

$$\text{So, ratio of their capitals} = (45000 \times 12) : (60000 \times 9) : (90000 \times 3) \\ = 540000 : 540000 : 270000 = 2 : 2 : 1$$

$$\therefore \text{Alfred's share} = \text{Rs.} \left(16500 \times \frac{2}{5} \right) = \text{Rs.} 6600;$$

$$\text{Peter's share} = \text{Rs.} \left(16500 \times \frac{2}{5} \right) = \text{Rs.} 6600;$$

$$\text{Ronald's share} = \text{Rs.} \left(16500 \times \frac{1}{5} \right) = \text{Rs.} 3300.$$

Ex. 3. A, B and C start a business each investing Rs. 20,000. After 5 months A withdrew Rs. 5000, B withdrew Rs. 4000 and C invests Rs. 6000 more. At the end of the year, a total profit of Rs. 69,900 was recorded. Find the share of each.

Sol. Ratio of the capitals of A, B and C

$$= 20000 \times 5 + 15000 \times 7 : 20000 \times 5 + 16000 \times 7 : 20000 \times 5 + 26000 \times 7 \\ = 205000 : 212000 : 282000 = 205 : 212 : 282.$$

$$\therefore \text{A's share} = \text{Rs.} \left(69900 \times \frac{205}{699} \right) = \text{Rs.} 20500;$$

$$\text{B's share} = \text{Rs.} \left(69900 \times \frac{212}{699} \right) = \text{Rs.} 21200;$$

$$\text{C's share} = \text{Rs.} \left(69900 \times \frac{282}{699} \right) = \text{Rs.} 28200.$$

Ex. 4. A, B and C enter into partnership. A invests 3 times as much as B invests and B invests two-third of what C invests. At the end of the year, the profit earned is Rs. 6600. What is the share of B?

Sol. Let C's capital = Rs. x. Then, B's capital = Rs. $\frac{2}{3}x$.

$$\text{A's capital} = \text{Rs.} \left(3 \times \frac{2}{3}x \right) = \text{Rs.} 2x.$$

$$\therefore \text{Ratio of their capitals} = 2x : \frac{2}{3}x : x = 6 : 2 : 3.$$

$$\text{Hence, B's share} = \text{Rs.} \left(6600 \times \frac{2}{11} \right) = \text{Rs.} 1200.$$

Ex. 5. Four milkmen rented a pasture. A grazed 24 cows for 3 months; B 10 cows for 5 months; C 35 cows for 4 months and D 21 cows for 3 months. If A's share of rent is Rs. 720, find the total rent of the field.

$$\text{Sol. Ratio of shares of A, B, C, D} = (24 \times 3) : (10 \times 5) : (35 \times 4) : (21 \times 3) \\ = 72 : 50 : 140 : 63.$$

$$\text{Let total rent be Rs. } x. \text{ Then, A's share} = \text{Rs.} \frac{72x}{325}.$$

$$\therefore \frac{72x}{325} = 720 \Leftrightarrow x = \frac{720 \times 325}{72} = 3250.$$

Hence, total rent of the field is Rs. 3250.

Ex. 6. A invested Rs. 76,000 in a business. After few months, B joined him with Rs. 57,000. At the end of the year, the total profit was divided between them in the ratio 2 : 1. After how many months did B join?

Sol. Suppose B joined after x months. Then, B's money was invested for (12 - x) months.

$$\therefore \frac{76000 \times 12}{57000 \times (12 - x)} = \frac{2}{1} \Leftrightarrow 912000 = 114000 (12 - x)$$

$$\Leftrightarrow 114 (12 - x) = 912 \Leftrightarrow (12 - x) = 8 \Leftrightarrow x = 4.$$

Hence, B joined after 4 months.

Ex. 7. A, B and C enter into a partnership by investing in the ratio of 3 : 2 : 4. After one year, B invests another Rs. 2,70,000 and C, at the end of 2 years, also invests Rs. 2,70,000. At the end of three years, profits are shared in the ratio of 3 : 4 : 5. Find the initial investment of each.

Sol. Let the initial investments of A, B and C be Rs. $3x$, Rs. $2x$ and Rs. $4x$ respectively. Then,

$$(3x \times 36) : [(2x \times 12) + (2x + 270000) \times 24] : [(4x \times 24) + (4x + 270000) \times 12] \\ = 3 : 4 : 5.$$

$$\Leftrightarrow 108x : (72x + 6480000) : (144x + 3240000) = 3 : 4 : 5$$

$$\therefore \frac{108x}{72x + 6480000} = \frac{3}{4} \quad \Leftrightarrow \quad 432x = 216x + 19440000$$

$$\Leftrightarrow 216x = 19440000 \quad \Leftrightarrow \quad x = 90000.$$

Hence, A's initial investment = $3x$ = Rs. 2,70,000;

B's initial investment = $2x$ = Rs. 1,80,000;

C's initial investment = $4x$ = Rs. 3,60,000.

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14. CHAIN RULE

IMPORTANT FACTS AND FORMULAE

1. **Direct Proportion** : Two quantities are said to be directly proportional, if on the increase (or decrease) of the one, the other increases (or decreases) to the same extent.

Ex. 1. Cost is directly proportional to the number of articles.

(More Articles, More Cost)

Ex. 2. Work done is directly proportional to the number of men working on it.

(More Men, More Work)

2. **Indirect Proportion** : Two quantities are said to be indirectly proportional, if on the increase of the one, the other decreases to the same extent and vice-versa.

Ex. 1. The time taken by a car in covering a certain distance is inversely proportional to the speed of the car.

(More speed, Less is the time taken to cover a distance)

Ex. 2. Time taken to finish a work is inversely proportional to the number of persons working at it.

(More persons, Less is the time taken to finish a job)

Remark : In solving questions by chain rule, we compare every item with the term to be found out.

SOLVED EXAMPLES

Ex. 1. If 15 toys cost Rs. 234, what do 35 toys cost ?

Sol. Let the required cost be Rs. x . Then,

More toys, More cost (Direct Proportion)

$$\therefore 15 : 35 :: 234 : x \Leftrightarrow (15 \times x) = (35 \times 234) \Leftrightarrow x = \left(\frac{35 \times 234}{15} \right) = 546.$$

Hence, the cost of 35 toys is Rs. 546.

Ex. 2. If 36 men can do a piece of work in 25 hours, in how many hours will 15 men do it ?

Sol. Let the required number of hours be x . Then,

Less men, More hours (Indirect Proportion)

$$\therefore 15 : 36 :: 25 : x \Leftrightarrow (15 \times x) = (36 \times 25) \Leftrightarrow x = \frac{36 \times 25}{15} = 60.$$

Hence, 15 men can do it in 60 hours.

Ex. 3. If the wages of 6 men for 15 days be Rs. 2100, then find the wages of 9 men for 12 days.

Sol. Let the required wages be Rs. x .

More men, More wages (Direct Proportion)

Less days, Less wages (Direct Proportion)

$$\left. \begin{array}{l} \text{Men } 6 : 9 \\ \text{Days } 15 : 12 \end{array} \right\} :: 2100 : x$$

$$\therefore (6 \times 15 \times x) = (9 \times 12 \times 2100) \Leftrightarrow x = \left(\frac{9 \times 12 \times 2100}{6 \times 15} \right) = 2520.$$

Hence, the required wages are Rs. 2520.

Ex. 4. If 20 men can build a wall 56 metres long in 6 days, what length of a similar wall can be built by 35 men in 3 days?

Sol. Let the required length be x metres.

More men, More length built (Direct Proportion)

Less days, Less length built (Direct Proportion)

$$\left. \begin{array}{l} \text{Men } 20 : 35 \\ \text{Days } 6 : 3 \end{array} \right\} \therefore 56 : x$$

$$\therefore (20 \times 6 \times x) = (35 \times 3 \times 56) \Leftrightarrow x = \frac{(35 \times 3 \times 56)}{120} = 49.$$

Hence, the required length is 49 m.

Ex. 5. If 15 men, working 9 hours a day, can reap a field in 16 days, in how many days will 18 men reap the field, working 8 hours a day?

Sol. Let the required number of days be x .

More men, Less days (Indirect Proportion)

Less hours per day, More days (Indirect Proportion)

$$\left. \begin{array}{l} \text{Men } 18 : 15 \\ \text{Hours per day } 8 : 9 \end{array} \right\} \therefore 16 : x$$

$$\therefore (18 \times 8 \times x) = (15 \times 9 \times 16) \Leftrightarrow x = \left(\frac{15 \times 144}{144} \right) = 15.$$

Hence, required number of days = 15.

Ex. 6. If 9 engines consume 24 metric tonnes of coal, when each is working 8 hours a day, how much coal will be required for 8 engines, each running 13 hours a day, being given that 3 engines of former type consume as much as 4 engines of latter type

Sol. Let 3 engines of former type consume 1 unit in 1 hour.

Then, 4 engines of latter type consume 1 unit in 1 hour.

$$\therefore 1 \text{ engine of former type consumes } \frac{1}{3} \text{ unit in 1 hour.}$$

$$1 \text{ engine of latter type consumes } \frac{1}{4} \text{ unit in 1 hour.}$$

Let the required consumption of coal be x units.

Less engines, Less coal consumed (Direct Proportion)

More working hours, More coal consumed (Direct Proportion)

Less rate of consumption, Less coal consumed (Direct Proportion)

$$\left. \begin{array}{l} \text{Number of engines } 9 : 8 \\ \text{Working hours } 8 : 13 \\ \text{Rate of consumption } \frac{1}{3} : \frac{1}{4} \end{array} \right\} \therefore 24 : x$$

$$\therefore \left(9 \times 8 \times \frac{1}{3} \times x \right) = \left(8 \times 13 \times \frac{1}{4} \times 24 \right) \Leftrightarrow 24x = 624 \Leftrightarrow x = 26.$$

Hence, the required consumption of coal = 26 metric tonnes.

Ex. 7. A contract is to be completed in 46 days and 117 men were set to work, each working 8 hours a day. After 33 days, $\frac{4}{7}$ of the work is completed. How many additional men may be employed so that the work may be completed in time, each man now working 9 hours a day?

Sol. Remaining work = $\left(1 - \frac{4}{7}\right) = \frac{3}{7}$. Remaining period = (46 - 33) days = 13 days.

Let the total men working at it be x .

Less work, Less men (Direct Proportion)

Less days, More men (Indirect Proportion)

More Hrs/Day, Less men (Indirect Proportion)

$$\left. \begin{array}{l} \text{Work} \quad \frac{4}{7} : \frac{3}{7} \\ \text{Days} \quad 13 : 33 \\ \text{Hrs/Day} \quad 9 : 8 \end{array} \right\} :: 117 : x$$

$$\therefore \frac{4}{7} \times 13 \times 9 \times x = \frac{3}{7} \times 33 \times 8 \times 117 \text{ or } x = \left(\frac{3 \times 33 \times 8 \times 117}{4 \times 13 \times 9} \right) = 198.$$

\therefore Additional men to be employed = (198 - 117) = 81.

Ex. 8. A garrison of 3300 men had provisions for 32 days, when given at the rate of 850 gms per head. At the end of 7 days, a reinforcement arrives and it was found that the provisions will last 17 days more, when given at the rate of 825 gms per head. What is the strength of the reinforcement?

Sol. The problem becomes :

3300 men taking 850 gms per head have provisions for (32 - 7) or 25 days. How many men taking 825 gms each have provisions for 17 days?

Less ration per head, more men (Indirect Proportion)

Less days, More men (Indirect Proportion)

$$\left. \begin{array}{l} \text{Ration } 825 : 850 \\ \text{Days } 17 : 25 \end{array} \right\} :: 3300 : x$$

$$\therefore 825 \times 17 \times x = 850 \times 25 \times 3300 \text{ or } x = \frac{850 \times 25 \times 3300}{825 \times 17} = 5500.$$

\therefore Strength of reinforcement = (5500 - 3300) = 1700.

15. TIME AND WORK

IMPORTANT FACTS AND FORMULAE

1. If A can do a piece of work in n days, then A's 1 day's work = $\frac{1}{n}$.
2. If A's 1 day's work = $\frac{1}{n}$, then A can finish the work in n days.
3. If A is thrice as good a workman as B, then :
Ratio of work done by A and B = 3 : 1.
Ratio of times taken by A and B to finish a work = 1 : 3.

SOLVED EXAMPLES

Ex. 1. Worker A takes 8 hours to do a job. Worker B takes 10 hours to do the same job. How long should it take both A and B, working together but independently, to do the same job? (IGNOU, 2003)

Sol. A's 1 hour's work = $\frac{1}{8}$, B's 1 hour's work = $\frac{1}{10}$.

$$(A + B)\text{'s 1 hour's work} = \left(\frac{1}{8} + \frac{1}{10}\right) = \frac{9}{40}.$$

\therefore Both A and B will finish the work in $\frac{40}{9} = 4\frac{4}{9}$ days.

Ex. 2. A and B together can complete a piece of work in 4 days. If A alone can complete the same work in 12 days, in how many days can B alone complete that work? (Bank P.O. 2003)

Sol. (A + B)'s 1 day's work = $\frac{1}{4}$, A's 1 day's work = $\frac{1}{12}$.

$$\therefore \text{B's 1 day's work} = \left(\frac{1}{4} - \frac{1}{12}\right) = \frac{1}{6}.$$

Hence, B alone can complete the work in 6 days.

Ex. 3. A can do a piece of work in 7 days of 9 hours each and B can do it in 6 days of 7 hours each. How long will they take to do it, working together $8\frac{2}{5}$ hours a day?

Sol. A can complete the work in $(7 \times 9) = 63$ hours.

B can complete the work in $(6 \times 7) = 42$ hours.

$$\therefore \text{A's 1 hour's work} = \frac{1}{63} \text{ and B's 1 hour's work} = \frac{1}{42}.$$

$$(A + B)\text{'s 1 hour's work} = \left(\frac{1}{63} + \frac{1}{42}\right) = \frac{5}{126}.$$

Both will finish the work in $\left(\frac{126}{5}\right)$ hrs.

$$\text{Number of days of } 8\frac{2}{5} \text{ hrs each} = \left(\frac{126}{5} \times \frac{5}{42}\right) = 3 \text{ days.}$$

Ex. 4. *A and B can do a piece of work in 18 days; B and C can do it in 24 days; A and C can do it in 36 days. In how many days will A, B and C finish it, working together and separately?*

Sol. $(A + B)$'s 1 day's work = $\frac{1}{18}$, $(B + C)$'s 1 day's work = $\frac{1}{24}$,

and $(A + C)$'s 1 day's work = $\frac{1}{36}$.

Adding, we get : $2(A + B + C)$'s 1 day's work = $\left(\frac{1}{18} + \frac{1}{24} + \frac{1}{36}\right) = \frac{9}{72} = \frac{1}{8}$.

$\therefore (A + B + C)$'s 1 day's work = $\frac{1}{16}$.

Thus, A, B and C together can finish the work in 16 days.

Now, A's 1 day's work = $[(A + B + C)$'s 1 day's work] - $[(B + C)$'s 1 day's work]

$$= \left(\frac{1}{16} - \frac{1}{24}\right) = \frac{1}{48}$$

\therefore A alone can finish the work in 48 days.

Similarly, B's 1 day's work = $\left(\frac{1}{16} - \frac{1}{36}\right) = \frac{5}{144}$.

\therefore B alone can finish the work in $\frac{144}{5} = 28\frac{4}{5}$ days.

And, C's 1 day's work = $\left(\frac{1}{16} - \frac{1}{18}\right) = \frac{1}{144}$.

\therefore C alone can finish the work in 144 days.

Ex. 5. *A is twice as good a workman as B and together they finish a piece of work in 18 days. In how many days will A alone finish the work?*

Sol. (A) 's 1 day's work : (B) 's 1 day's work = 2 : 1.

$(A + B)$'s 1 day's work = $\frac{1}{18}$.

Divide $\frac{1}{18}$ in the ratio 2 : 1.

A's 1 day's work = $\left(\frac{1}{18} \times \frac{2}{3}\right) = \frac{1}{27}$.

Hence, A alone can finish the work in 27 days.

Ex. 6. *A can do a certain job in 12 days. B is 60% more efficient than A. How many days does B alone take to do the same job?*

Sol. Ratio of times taken by A and B = 160 : 100 = 8 : 5.

Suppose B alone takes x days to do the job.

Then, $8 : 5 :: 12 : x \Rightarrow 8x = 5 \times 12 \Rightarrow x = 7\frac{1}{2}$ days.

Ex. 7. *A can do a piece of work in 80 days. He works at it for 10 days and then B alone finishes the remaining work in 42 days. In how much time will A and B, working together, finish the work?*

Sol. Work done by A in 10 days = $\left(\frac{1}{80} \times 10\right) = \frac{1}{8}$.

$$\text{Remaining work} = \left(1 - \frac{1}{8}\right) = \frac{7}{8}.$$

Now, $\frac{7}{8}$ work is done by B in 42 days.

Whole work will be done by B in $\left(42 \times \frac{8}{7}\right) = 48$ days.

$$\therefore \text{A's 1 day's work} = \frac{1}{80} \text{ and B's 1 day's work} = \frac{1}{48}.$$

$$\therefore (\text{A} + \text{B})\text{'s 1 day's work} = \left(\frac{1}{80} + \frac{1}{48}\right) = \frac{8}{240} = \frac{1}{30}.$$

Hence, both will finish the work in 30 days.

Ex. 8. A and B undertake to do a piece of work for Rs. 600. A alone can do it 6 days while B alone can do it in 8 days. With the help of C, they finish it in 3 days. Find the share of each.

$$\text{Sol. C's 1 day's work} = \frac{1}{3} - \left(\frac{1}{6} + \frac{1}{8}\right) = \frac{1}{24}.$$

$$\therefore \text{A : B : C} = \text{Ratio of their 1 day's work} = \frac{1}{6} : \frac{1}{8} : \frac{1}{24} = 4 : 3 : 1.$$

$$\therefore \text{A's share} = \text{Rs.} \left(600 \times \frac{4}{8}\right) = \text{Rs. 300}, \text{ B's share} = \text{Rs.} \left(600 \times \frac{3}{8}\right) = \text{Rs. 225}.$$

$$\text{C's share} = \text{Rs. } [600 - (300 + 225)] = \text{Rs. 75}.$$

Ex. 9. A and B working separately can do a piece of work in 9 and 12 days respectively. If they work for a day alternately, A beginning, in how many days, the work will be completed?

$$\text{Sol. (A + B)'s 2 days' work} = \left(\frac{1}{9} + \frac{1}{12}\right) = \frac{7}{36}.$$

$$\text{Work done in 5 pairs of days} = \left(5 \times \frac{7}{36}\right) = \frac{35}{36}.$$

$$\text{Remaining work} = \left(1 - \frac{35}{36}\right) = \frac{1}{36}.$$

On 11th day, it is A's turn. $\frac{1}{9}$ work is done by him in 1 day.

$$\frac{1}{36} \text{ work is done by him in } \left(9 \times \frac{1}{36}\right) = \frac{1}{4} \text{ day.}$$

$$\therefore \text{Total time taken} = \left(10 + \frac{1}{4}\right) \text{ days} = 10\frac{1}{4} \text{ days.}$$

Ex. 10. 45 men can complete a work in 16 days. Six days after they started working 10 more men joined them. How many days will they now take to complete the remaining work?

Sol. (45 × 16) men can complete the work in 1 day.

$$\therefore 1 \text{ man's 1 day's work} = \frac{1}{720}.$$

$$45 \text{ men's 6 days' work} = \left(\frac{1}{16} \times 6\right) = \frac{3}{8}. \text{ Remaining work} = \left(1 - \frac{3}{8}\right) = \frac{5}{8}.$$

$$75 \text{ men's 1 day's work} = \frac{75}{720} = \frac{5}{48}.$$

Now, $\frac{5}{48}$ work is done by them in 1 day.

$\therefore \frac{5}{8}$ work is done by them in $\left(\frac{48}{5} \times \frac{5}{8}\right) = 6$ days.

Ex. 11. 2 men and 3 boys can do a piece of work in 10 days while 3 men and 2 boys can do the same work in 8 days. In how many days can 2 men and 1 boy do the work?

Sol. Let 1 man's 1 day's work = x and 1 boy's 1 day's work = y .

Then, $2x + 3y = \frac{1}{10}$ and $3x + 2y = \frac{1}{8}$.

Solving, we get : $x = \frac{7}{200}$ and $y = \frac{1}{100}$.

\therefore (2 men + 1 boy)'s 1 day's work = $\left(2 \times \frac{7}{200} + 1 \times \frac{1}{100}\right) = \frac{16}{200} = \frac{2}{25}$.

So, 2 men and 1 boy together can finish the work in $\frac{25}{2} = 12\frac{1}{2}$ days.

16. PIPES AND CISTERNS

IMPORTANT FACTS AND FORMULAE

1. Inlet: A pipe connected with a tank or a cistern or a reservoir, that fills it, is known as an inlet.

Outlet: A pipe connected with a tank or a cistern or a reservoir, emptying it, is known as an outlet.

2. (i) If a pipe can fill a tank in x hours, then : part filled in 1 hour = $1/x$

(ii) If a pipe can empty a full tank in y hours, then : part emptied in 1 hour = $1/y$

(iii) If a pipe can fill a tank in x hours and another pipe can empty the full tank in y hours (where $y > x$), then on opening both the pipes, the net part filled in 1 hour = $(1/x) - (1/y)$

(iv) If a pipe can fill a tank in x hours and another pipe can empty the full tank in y hours (where $x > y$), then on opening both the pipes, the net part emptied in 1 hour = $(1/y) - (1/x)$

SOLVED EXAMPLES

Ex. 1: Two pipes A and B can fill a tank in 36 hours and 46 hours respectively. If both the pipes are opened simultaneously, how much time will be taken to fill the tank?

Sol: Part filled by A in 1 hour = $(1/36)$;

Part filled by B in 1 hour = $(1/45)$;

Part filled by (A + B) In 1 hour = $(1/36) + (1/45) = (9/180) = (1/20)$

Hence, both the pipes together will fill the tank in 20 hours.

Ex. 2: Two pipes can fill a tank in 10 hours and 12 hours respectively while a third, pipe empties the full tank in 20 hours. If all the three pipes operate simultaneously, in how much time will the tank be filled?

Sol: Net part filled In 1 hour = $(1/10) + (1/12) - (1/20) = (8/60) = (2/15)$.

The tank will be full in $\frac{15}{2}$ hrs = 7 hrs 30 min.

Ex. 3: If two pipes function simultaneously, the reservoir will be filled in 12 hours. One pipe fills the reservoir 10 hours faster than the other. How many hours does it take the second pipe to fill the reservoir?

Sol: let the reservoir be filled by first pipe in x hours.

Then, second pipe fill it in $(x+10)$ hrs.

Therefore $(\frac{1}{x}) + (\frac{1}{x+10}) = (\frac{1}{12}) \Leftrightarrow (x+10+x)/(x(x+10)) = (\frac{1}{12})$.

$$\Leftrightarrow x^2 - 14x - 120 = 0 \Leftrightarrow (x-20)(x+6) = 0$$

$$\Leftrightarrow x = 20 \quad [\text{neglecting the negative value of } x]$$

so, the second pipe will take $(20+10)$ hrs. (i.e) 30 hours to fill the reservoir

Ex. 4: A cistern has two taps which fill it in 12 minutes and 15 minutes respectively. There is also a waste pipe in the cistern. When all the 3 are opened, the empty cistern is full in 20 minutes. How long will the waste pipe take to empty the full cistern?

Sol: Work done by the waste pipe in 1 min

$$= (\frac{1}{20}) - (\frac{1}{12}) + (\frac{1}{15}) = -\frac{1}{10} \quad [\text{negative sign means emptying}]$$

therefore the waste pipe will empty the full cistern in 10 min

Ex. 5: An electric pump can fill a tank in 3 hours. Because of a leak in, the tank it took $3\frac{1}{2}$ hours to fill the tank. If the tank is full, how much time will the leak take to empty it?

Sol: work done by the leak in 1 hour $= (\frac{1}{3}) - (\frac{1}{(7/2)}) = (\frac{1}{3}) - (\frac{2}{7}) = (\frac{1}{21})$.

The leak will empty the tank in 21 hours.

Ex. 6. Two pipes can fill a cistern in 14 hours and 16 hours respectively. The pipes are opened simultaneously and it is found that due to leakage in the bottom it took 32 minutes more to fill the cistern. When the cistern is full, in what time will the leak empty it?

Sol: Work done by the two pipes in 1 hour $= (1/14) + (1/16) = (15/112)$.

Time taken by these pipes to fill the tank $= (112/15)$ hrs $= 7$ hrs 28 min.

Due to leakage, time taken $= 7$ hrs 28 min $+ 32$ min $= 8$ hrs

Work done by (two pipes + leak) in 1 hour $= (1/8)$.

Work done by the leak in 1 hour $= (15/112) - (1/8) = (1/112)$.

Leak will empty the full cistern in 112 hours.

Ex. 7: Two pipes A and B can fill a tank in 36 min. and 45 min. respectively. A water pipe C can empty the tank in 30 min. First A and B are opened. after 7 min, C is also opened. In how much time, the tank is full?

Sol: Part filled in 7 min. $= 7 * ((1/36) + (1/45)) = (7/20)$.

Remaining part $= (1 - (7/20)) = (13/20)$.

Net part filled in 1 min. when A, B and C are opened $= (1/36) + (1/45) - (1/30) = (1/60)$.

Now, $(1/60)$ part is filled in one minute.

$(13/20)$ part is filled in $(60 * (13/20)) = 39$ minutes.

Ex.8: Two pipes A,B can fill a tank in 24 min. and 32 min. respectively. If both the pipes are opened simultaneously, after how much time B should be closed so that the tank is full in 18 min.?

Sol: let B be closed after x min. then ,

Part filled by (A+B) in x min. + part filled by A in $(18-x)$ min. $= 1$

Therefore $x * ((1/24) + (1/32)) + (18-x) * (1/24) = 1 \Leftrightarrow (7x/96) + ((18-x)/24) = 1$.

$\Leftrightarrow 7x + 4 * (18-x) = 96$.

Hence, B must be closed after 8 min.

17. TIME AND DISTANCE

IMPORTANT FACTS AND FORMULAE

$$1. \text{ Speed} = \left(\frac{\text{Distance}}{\text{Time}} \right), \text{ Time} = \left(\frac{\text{Distance}}{\text{Speed}} \right), \text{ Distance} = (\text{Speed} * \text{Time})$$

$$2. x \text{ km / hr} = x * \frac{5}{18}$$

$$3. x \text{ m/sec} = (x * 18/5) \text{ km /hr}$$

4. If the ratio of the speeds of A and B is a:b , then the ratio of the times taken by them to cover the same distance is $\frac{1}{a} : \frac{1}{b}$

a b

or b:a.

5. Suppose a man covers a certain distance at x km/ hr and an equal distance at y km / hr . Then , the average speed during the whole journey is $\frac{2xy}{x+y}$ km/ hr.

SOLVED EXAMPLES

Ex. 1. How many minutes does Aditya take to cover a distance of 400 m, if he runs at a speed of 20 km/hr?

$$\text{Sol. Aditya's speed} = 20 \text{ km/hr} = \left\{ 20 * \frac{5}{18} \right\} \text{ m/sec} = \frac{50}{9} \text{ m/sec}$$

$$\therefore \text{Time taken to cover 400 m} = \left\{ 400 * \frac{9}{50} \right\} \text{ sec} = 72 \text{ sec} = 1 \frac{12}{60} \text{ min} = 1 \frac{1}{5} \text{ min.}$$

Ex. 2. A cyclist covers a distance of 750 m in 2 min 30 sec. What is the speed in km/hr of the cyclist?

$$\text{Sol. Speed} = \left\{ \frac{750}{150} \right\} \text{ m/sec} = 5 \text{ m/sec} = \left\{ 5 * \frac{18}{5} \right\} \text{ km/hr} = 18 \text{ km/hr}$$

Ex. 3. A dog takes 4 leaps for every 5 leaps of a hare but 3 leaps of a dog are equal to 4 leaps of the hare. Compare their speeds.

Sol. Let the distance covered in 1 leap of the dog be x and that covered in 1 leap of the hare be y.

$$\text{Then , } 3x = 4y \Rightarrow x = \frac{4}{3} y \Rightarrow 4x = \frac{16}{3} y.$$

\therefore Ratio of speeds of dog and hare = Ratio of distances covered by them in the same time

$$= 4x : 5y = \frac{16}{3} y : 5y = \frac{16}{3} : 5 = 16:15$$

Ex. 4. While covering a distance of 24 km, a man noticed that after walking for 1 hour and 40 minutes, the distance covered by him was $\frac{5}{7}$ of the remaining distance. What was his speed in metres per second?

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Sol. Let the speed be x km/hr.

Then, distance covered in 1 hr. 40 min. i.e., $1\frac{2}{3}$ hrs = $\frac{5x}{3}$ km

Remaining distance = $\{24 - \frac{5x}{3}\}$ km.

$$\therefore \frac{5x}{3} = \frac{5}{7} \left\{24 - \frac{5x}{3}\right\} \Leftrightarrow \frac{5x}{3} = \frac{5}{7} \left\{\frac{72-5x}{3}\right\} \Leftrightarrow 7x = 72 - 5x$$

$$\Leftrightarrow 12x = 72 \Leftrightarrow x = 6$$

$$\text{Hence speed} = 6 \text{ km/hr} = \left\{6 \times \frac{5}{18}\right\} \text{ m/sec} = \frac{5}{3} \text{ m/sec} = 1\frac{2}{3}$$

Ex. 5. Peter can cover a certain distance in 1 hr. 24 min. by covering two-third of the distance at 4 kmph and the rest at 5 kmph. Find the total distance.

Sol. Let the total distance be x km. Then,

$$\frac{\frac{2}{3}x}{4} + \frac{\frac{1}{3}x}{5} = \frac{7}{5} \Leftrightarrow \frac{x}{6} + \frac{x}{15} = \frac{7}{5} \Leftrightarrow 7x = 42 \Leftrightarrow x = 6$$

Ex. 6. A man traveled from the village to the post-office at the rate of 25 kmph and walked back at the rate of 4 kmph. If the whole journey took 5 hours 48 minutes, find the distance of the post-office from the village.

Sol. Average speed = $\left\{\frac{2xy}{x+y}\right\}$ km/hr = $\left\{\frac{2 \times 25 \times 4}{25+4}\right\}$ km/hr = $\frac{200}{29}$ km/hr

Distance traveled in 5 hours 48 minutes i.e., $5\frac{4}{5}$ hrs. = $\left\{\frac{200}{29} \times \frac{29}{5}\right\}$ km = 40 km

$$\text{Distance of the post-office from the village} = \left\{\frac{40}{2}\right\} = 20 \text{ km}$$

Ex. 7. An aeroplane flies along the four sides of a square at the speeds of 200, 400, 600 and 800 km/hr. Find the average speed of the plane around the field.

Sol. :

Let each side of the square be x km and let the average speed of the plane around the field be y km per hour then ,

$$\frac{x}{200} + \frac{x}{400} + \frac{x}{600} + \frac{x}{800} = \frac{4x}{y} \Leftrightarrow \frac{25x}{2500} \Leftrightarrow \frac{4x}{y} \Leftrightarrow y = \left(\frac{2400 \times 4}{25}\right) = 384$$

hence average speed = 384 km/hr

Ex. 8. Walking at $\frac{5}{7}$ of its usual speed, a train is 10 minutes too late. Find its usual time to cover the journey.

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Sol. : New speed = $\frac{5}{6}$ of the usual speed

New time taken = $\frac{6}{5}$ of the usual time

So, $(\frac{6}{5} \text{ of the usual time }) - (\text{usual time}) = 10 \text{ minutes.}$

$\Rightarrow \frac{1}{5} \text{ of the usual time} = 10 \text{ minutes.}$

$\Rightarrow \text{usual time} = 10 \text{ minutes}$

Ex. 9. If a man walks at the rate of 5 kmph, he misses a train by 7 minutes. However, if he walks at the rate of 6 kmph, he reaches the station 5 minutes before the arrival of the train. Find the distance covered by him to reach the station.

Sol. Let the required distance be x km

Difference in the time taken at two speeds = 1 min = $\frac{1}{2}$ hr

Hence $\frac{x}{5} - \frac{x}{6} = \frac{1}{5} \Leftrightarrow 6x - 5x = 6$

$\Leftrightarrow x = 6$

Hence, the required distance is 6 km

Ex. 10. A and B are two stations 390 km apart. A train starts from A at 10 a.m. and travels towards B at 65 kmph. Another train starts from B at 11 a.m. and travels towards A at 35 kmph. At what time do they meet?

Sol. Suppose they meet x hours after 10 a.m. Then,

(Distance moved by first in x hrs) + [Distance moved by second in $(x-1)$ hrs] = 390.

$$65x + 35(x-1) = 390 \Rightarrow 100x = 425 \Rightarrow x = 17/4$$

So, they meet 4 hrs.15 min. after 10 a.m i.e., at 2.15 p.m.

Ex. 11. A goods train leaves a station at a certain time and at a fixed speed. After \wedge hours, an express train leaves the same station and moves in the same direction at a uniform speed of 90 kmph. This train catches up the goods train in 4 hours. Find the speed of the goods train.

Sol. Let the speed of the goods train be x kmph.

Distance covered by goods train in 10 hours = Distance covered by express train in 4 hours

$$10x = 4 \times 90 \text{ or } x = 36.$$

So, speed of goods train = 36 kmph.

Ex. 12. A thief is spotted by a policeman from a distance of 100 metres. When the policeman starts the chase, the thief also starts running. If the speed of the thief be 8 km/hr and that of the policeman 10 km/hr, how far the thief will have run before he is overtaken?

Sol. Relative speed of the policeman = $(10-8) \text{ km/hr} = 2 \text{ km/hr.}$

Time taken by police man to cover 100m $\left(\frac{100}{1000} \times \frac{1}{2} \right) \text{ hr} = \frac{1}{20} \text{ hr.}$

In $\frac{1}{20}$ hrs, the thief covers a distance of $8 \times \frac{1}{20} \text{ km} = \frac{2}{5} \text{ km} = 400 \text{ m}$

Ex.13. I walk a certain distance and ride back taking a total time of 37 minutes. I could walk both ways in 55 minutes. How long would it take me to ride both ways?

Sol. Let the distance be x km. Then,

$$(\text{Time taken to walk } x \text{ km}) + (\text{time taken to ride } x \text{ km}) = 37 \text{ min.}$$

$$(\text{Time taken to walk } 2x \text{ km}) + (\text{time taken to ride } 2x \text{ km}) = 74 \text{ min.}$$

But, the time taken to walk $2x$ km = 55 min.

$$\text{Time taken to ride } 2x \text{ km} = (74 - 55) \text{ min} = 19 \text{ min.}$$

18. PROBLEMS ON TRAINS

IMPORTANT FACTS AND FORMULAE

1. $a \text{ km/hr} = (a * \frac{5}{18}) \text{ m/s}$.
2. $a \text{ m/s} = (\frac{a * 18}{5}) \text{ km/hr}$.
- 3 Time taken by a train of length l metres to pass a pole or a standing man or a signal post is equal to the time taken by the train to cover l metres.
4. Time taken by a train of length l metres to pass a stationary object of length b metres is the time taken by the train to cover $(l + b)$ metres.
5. Suppose two trains or two bodies are moving in the same direction at $u \text{ m/s}$ and $v \text{ m/s}$, where $u > v$, then their relative speed = $(u - v) \text{ m/s}$.
6. Suppose two trains or two bodies are moving in opposite directions at $u \text{ m/s}$ and $v \text{ m/s}$, then their relative speed is = $(u + v) \text{ m/s}$.
7. If two trains of length a metres and b metres are moving in opposite directions at $u \text{ m/s}$ and $v \text{ m/s}$, then time taken by the trains to cross each other = $\frac{(a + b)}{(u + v)}$ sec.
8. If two trains of length a metres and b metres are moving in the same direction at $u \text{ m/s}$ and $v \text{ m/s}$, then the time taken by the faster train to cross the slower train = $\frac{(a + b)}{(u - v)}$ sec.
9. If two trains (or bodies) start at the same time from points A and B towards each other and after crossing they take a and b sec in reaching B and A respectively, then
 $(A's \text{ speed}) : (B's \text{ speed}) = (b^{1/2} : a^{1/2})$.

SOLVED EXAMPLES

Ex.I. A train 100 m long is running at the speed of 30 km / hr. Find the time taken by it to pass a man standing near the railway line. (S.S.C. 2001)

Sol. Speed of the train = $(30 \times \frac{5}{18})$ m / sec = $(\frac{25}{3})$ m/ sec.

Distance moved in passing the standing man = 100 m.

Required time taken = $\frac{100}{(25/3)} = (100 \times \frac{3}{25})$ sec = 12 sec

Ex. 2. A train is moving at a speed of 132 km/hr. If the length of the train is 110 metres, how long will it take to cross a railway platform 165 metres long? (Section Officers', 2003)

Sol. Speed of train = $132 \times (\frac{5}{18})$ m/sec = $110/3$ m/sec.

Distance covered in passing the platform = $(110 + 165)$ m = 275 m.

Time taken = $275 \times (\frac{3}{110})$ sec = $15/2$ sec = $7 \frac{1}{2}$ sec

Ex. 3. A man is standing on a railway bridge which is 180 m long. He finds that a train crosses the bridge in 20 seconds but himself in 8 seconds. Find the length of the train and its speed?

Sol. Let the length of the train be x metres,

Then, the train covers x metres in 8 seconds and $(x + 180)$ metres in 20 sec

$$x/8 = (x+180)/20 \Leftrightarrow 20x = 8(x+180) \quad \Leftrightarrow \quad x = 120.$$

Length of the train = 120 m.

Speed of the train = $(120/8)$ m / sec = m / sec = $(15 \times \frac{18}{5})$ kmph = 54 km

Ex. 4. A train 150 m long is running with a speed of 68 kmph. In what time will it pass a man who is running at 8 kmph in the same direction in which the train is going?

Sol: Speed of the train relative to man = $(68 - 8)$ kmph

$$= (60 * \frac{5}{18}) \text{ m/sec} = \frac{50}{3} \text{ m/sec}$$

Time taken by the train to cross the man I

$$= \text{Time taken by It to cover } 150 \text{ m at } \frac{50}{3} \text{ m/sec} = 150 * \frac{3}{50} \text{ sec} = 9 \text{ sec}$$

Ex. 5. A train 220 m long is running with a speed of 59 kmph.. In what will it pass a man who is running at 7 kmph in the direction opposite to that in which the train is going?

sol. Speed of the train relative to man = $(59 + 7)$ kmph

$$= 66 * \frac{5}{18} \text{ m/sec} = \frac{55}{3} \text{ m/sec.}$$

Time taken by the train to cross the man = Time taken by it to cover 220 m at $\frac{55}{3} \text{ m/sec} = (220 * \frac{3}{55}) \text{ sec} = 12 \text{ sec}$

Ex. 6. Two trains 137 metres and 163 metres in length are running towards each other on parallel lines, one at the rate of 42 kmph and another at 48 kmph. In what time will they be clear of each other from the moment they meet?

Sol. Relative speed of the trains = $(42 + 48)$ kmph = 90 kmph

$$= (90 * \frac{5}{18}) \text{ m/sec} = 25 \text{ m/sec.}$$

Time taken by the trains to pass each other

$$= \text{Time taken to cover } (137 + 163) \text{ m at } 25 \text{ m/sec} = \frac{300}{25} \text{ sec} = 12 \text{ sec}$$

Ex. 7. Two trains 100 metres and 120 metres long are running in the same direction with speeds of 72 km/hr, In how much time will the first train cross the second?

Sol: Relative speed of the trains = $(72 - 54)$ km/hr = 18 km/hr

$$= (18 * \frac{5}{18}) \text{ m/sec} = 5 \text{ m/sec.}$$

Time taken by the trains to cross each other

$$= \text{Time taken to cover } (100 + 120) \text{ m at } 5 \text{ m/sec} = \frac{220}{5} \text{ sec} = 44 \text{ sec.}$$

Ex. 8. A train 100 metres long takes 6 seconds to cross a man walking at 5 kmph in the direction opposite to that of the train. Find the speed of the train.?

Sol: Let the speed of the train be x kmph.

Speed of the train relative to man = $(x + 5)$ kmph = $(x + 5) * 5/18$ m/sec.

Therefore $100 / ((x+5) * 5/18) = 6 \Leftrightarrow 30(x + 5) = 1800 \Leftrightarrow x = 55$

Speed of the train is 55 kmph.

Ex9. A train running at 54 kmph takes 20 seconds to pass a platform. Next it takes 12 sec to pass a man walking at 6 kmph in the same direction in which the train is going. Find the length of the train and the length of the platform.

Sol: Let the length of train be x metres and length of platform be y metres.

Speed of the train relative to man = $(54 - 6)$ kmph = 48 kmph

= $48 * (5/18)$ m/sec = $40/3$ m/sec.

In passing a man, the train covers its own length with relative speed.

Length of train = (Relative speed * Time) = $(\frac{40}{3}) * 12$ m = 160 m.

Also, speed of the train = $54 * (5/18)$ m / sec = 15 m / sec.

$(x+y)/15 = 20 \Leftrightarrow x + y = 300 \Leftrightarrow Y = (300 - 160)$ m = 140 m.

Ex10. A man sitting in a train which is traveling at 50 kmph observes that a goods train, traveling in opposite direction, takes 9 seconds to pass him. If the goods train is 280 m long, find its speed.?

Sol: Relative speed = $280/9$ m / sec = $((280/9) * (18/5))$ kmph = 112 kmph.

Speed of goods train = $(112 - 50)$ kmph = 62 kmph.

19. BOATS AND STREAMS

IMPORTANT FACTS AND FORMULAE

1. In water, the direction along the stream is called **downstream**. And, the direction against the stream is called **upstream**.
2. If the speed of a boat in still water is u km/hr and the speed of the stream is v km/hr, then :

$$\text{Speed downstream} = (u + v) \text{ km/hr}$$

$$\text{Speed upstream} = (u - v) \text{ km/hr.}$$

3. If the speed downstream is a km/hr and the speed upstream is b km/hr, then :

$$\text{Speed in still water} = \frac{1}{2}(a + b) \text{ km/hr}$$

$$\text{Rate of stream} = \frac{1}{2}(a - b) \text{ km/hr}$$

SOLVED EXAMPLES

Ex. 1. A man can row upstream at 7 kmph and downstream at 10 kmph. Find man's rate in still water and the rate of current.

Sol. Rate in still water = $\frac{1}{2}(10 + 7) \text{ km/hr} = 8.5 \text{ km/hr.}$

Rate of current = $\frac{1}{2}(10 - 7) \text{ km/hr} = 1.5 \text{ km/hr.}$

Ex. 2. A man takes 3 hours 45 minutes to row a boat 15 km downstream of a river and 2 hours 30 minutes to cover a distance of 5 km upstream. Find the speed of the river current in km/hr.

Sol. Rate downstream = $\left(\frac{15}{3\frac{3}{4}}\right) \text{ km/hr} = \left(15 \times \frac{4}{15}\right) \text{ km/hr} = 4 \text{ km/hr.}$

Rate upstream = $\left(\frac{5}{2\frac{1}{2}}\right) \text{ km/hr} = \left(5 \times \frac{2}{5}\right) \text{ km/hr} = 2 \text{ km/hr.}$

\therefore Speed of current = $\frac{1}{2}(4 - 2) \text{ km/hr} = 1 \text{ km/hr.}$

Ex. 3. A man can row 18 kmph in still water. It takes him thrice as long to row up as to row down the river. Find the rate of stream.

Sol. Let man's rate upstream be x kmph. Then, his rate downstream = $3x$ kmph.

\therefore Rate in still water = $\frac{1}{2}(3x + x) \text{ kmph} = 2x \text{ kmph.}$

So, $2x = 18$ or $x = 9$.

\therefore Rate upstream = 9 km/hr, Rate downstream = 27 km/hr.

Hence, rate of stream = $\frac{1}{2}(27 - 9) \text{ km/hr} = 9 \text{ km/hr.}$

Ex. 4. There is a road beside a river. Two friends started from a place A, moved to a temple situated at another place B and then returned to A again. One of them moves on a cycle at a speed of 12 km/hr, while the other sails on a boat at a speed of 10 km/hr. If the river flows at the speed of 4 km/hr, which of the two friends will return to place A first?

(R.R.B. 2001)

Sol. Clearly, the cyclist moves both ways at a speed of 12 km/hr.

So, average speed of the cyclist = 12 km/hr.

The boat sailor moves downstream @ $(10 + 4)$ i.e., 14 km/hr and upstream @ $(10 - 4)$ i.e., 6 km/hr.

$$\begin{aligned}\text{So, average speed of the boat sailor} &= \left(\frac{2 \times 14 \times 6}{14 + 6} \right) \text{ km/hr} \\ &= \frac{42}{5} \text{ km/hr} = 8.4 \text{ km/hr.}\end{aligned}$$

Since the average speed of the cyclist is greater, he will return to A first.

Ex. 5. A man can row $7\frac{1}{2}$ kmph in still water. If in a river running at 1.5 km an hour, it takes him 50 minutes to row to a place and back, how far off is the place?

(R.R.B. 2002)

Sol. Speed downstream = $(7.5 + 1.5)$ kmph = 9 kmph;

Speed upstream = $(7.5 - 1.5)$ kmph = 6 kmph.

Let the required distance be x km. Then,

$$\frac{x}{9} + \frac{x}{6} = \frac{50}{60} \Leftrightarrow 2x + 3x = \left(\frac{5}{6} \times 18 \right) \Leftrightarrow 5x = 15 \Leftrightarrow x = 3.$$

Hence, the required distance is 3 km.

Ex. 6. In a stream running at 2 kmph, a motorboat goes 6 km upstream and back again to the starting point in 33 minutes. Find the speed of the motorboat in still water.

Sol. Let the speed of the motorboat in still water be x kmph. Then,

Speed downstream = $(x + 2)$ kmph; Speed upstream = $(x - 2)$ kmph.

$$\begin{aligned}\therefore \frac{6}{x+2} + \frac{6}{x-2} &= \frac{33}{60} \Leftrightarrow 11x^2 - 240x - 44 = 0 \Leftrightarrow 11x^2 - 242x + 2x - 44 = 0 \\ &\Leftrightarrow (x - 22)(11x + 2) = 0 \Leftrightarrow x = 22.\end{aligned}$$

Hence, speed of motorboat in still water = 22 kmph.

Ex. 7. A man can row 40 km upstream and 55 km downstream in 13 hours. Also, he can row 30 km upstream and 44 km downstream in 10 hours. Find the speed of the man in still water and the speed of the current.

Sol. Let rate upstream = x km/hr and rate downstream = y km/hr.

$$\text{Then, } \frac{40}{x} + \frac{55}{y} = 13 \quad \dots(i) \quad \text{and} \quad \frac{30}{x} + \frac{44}{y} = 10 \quad \dots(ii)$$

Multiplying (ii) by 4 and (i) by 3 and subtracting, we get : $\frac{11}{y} = 1$ or $y = 11$.

Substituting $y = 11$ in (i), we get : $x = 5$.

$$\therefore \text{Rate in still water} = \frac{1}{2}(11 + 5) \text{ kmph} = 8 \text{ kmph.}$$

$$\text{Rate of current} = \frac{1}{2}(11 - 5) \text{ kmph} = 3 \text{ kmph.}$$

20. ALLIGATION OR MIXTURE

IMPORTANT FACTS AND FORMULAE

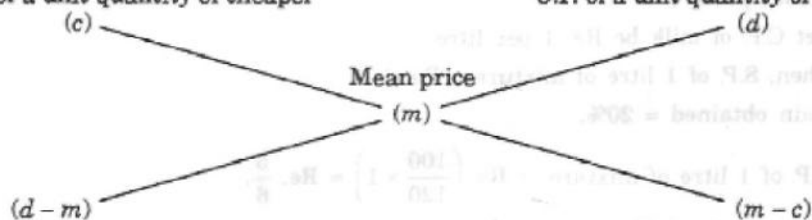
1. **Alligation** : It is the rule that enables us to find the ratio in which two or more ingredients at the given price must be mixed to produce a mixture of a desired price.
2. **Mean Price** : The cost price of a unit quantity of the mixture is called the mean price.
3. **Rule of Alligation** : If two ingredients are mixed, then

$$\left(\frac{\text{Quantity of cheaper}}{\text{Quantity of dearer}} \right) = \frac{(\text{C.P. of dearer}) - (\text{Mean price})}{(\text{Mean price}) - (\text{C.P. of cheaper})}$$

We present as under :

C.P. of a unit quantity of cheaper

C.P. of a unit quantity of dearer



$$\therefore (\text{Cheaper quantity}) : (\text{Dearer quantity}) = (d - m) : (m - c)$$

4. Suppose a container contains x units of liquid from which y units are taken out and replaced by water. After n operations, the quantity of pure liquid = $\left[x \left(1 - \frac{y}{x} \right)^n \right]$ units.

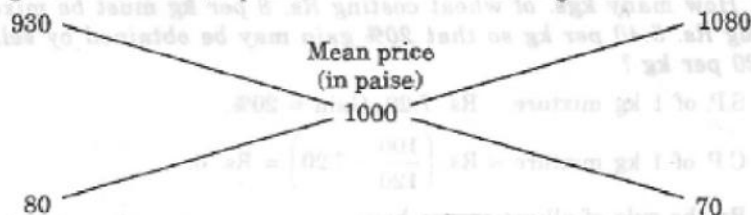
SOLVED EXAMPLES

Ex. 1. In what ratio must rice at Rs. 9.30 per kg be mixed with rice at Rs. 10.80 per kg so that the mixture be worth Rs. 10 per kg ?

Sol. By the rule of alligation, we have :

C.P. of 1 kg rice of 1st kind (in paise)

C.P. of 1 kg rice of 2nd kind (in paise)



\therefore Required ratio = 80 : 70 = 8 : 7.

Ex. 2. How much water must be added to 60 litres of milk at $1\frac{1}{2}$ litres for Rs. 20 so as to have a mixture worth Rs. $10\frac{2}{3}$ a litre ?

Sol. C.P. of 1 litre of milk = Rs. $\left(20 \times \frac{2}{3} \right)$ = Rs. $\frac{40}{3}$.

C.P. of 1 litre of water
0

C.P. of 1 litre of milk
Rs. $\frac{40}{3}$

Mean price
 $\left(\text{Rs. } \frac{32}{3}\right)$

$\left(\frac{40}{3} - \frac{32}{3}\right) = \frac{8}{3}$

$\left(\frac{32}{3} - 0\right) = \frac{32}{3}$

\therefore Ratio of water and milk = $\frac{8}{3} : \frac{32}{3} = 8 : 32 = 1 : 4$.

\therefore Quantity of water to be added to 60 litres of milk = $\left(\frac{1}{4} \times 60\right)$ litres = 15 litres.

Ex. 3. In what ratio must water be mixed with milk to gain 20% by selling the mixture at cost price?

Sol. Let C.P. of milk be Re. 1 per litre.

Then, S.P. of 1 litre of mixture = Re. 1.

Gain obtained = 20%.

\therefore C.P. of 1 litre of mixture = Rs. $\left(\frac{100}{120} \times 1\right) = \text{Re. } \frac{5}{6}$.

By the rule of alligation, we have :

C.P. of 1 litre of water
0

C.P. of 1 litre of milk
Re. 1

Mean price
 $\left(\text{Re. } \frac{5}{6}\right)$

$\left(1 - \frac{5}{6}\right) = \frac{1}{6}$

$\left(\frac{5}{6} - 0\right) = \frac{5}{6}$

\therefore Ratio of water and milk = $\frac{1}{6} : \frac{5}{6} = 1 : 5$.

Ex. 4. How many kgs. of wheat costing Rs. 8 per kg must be mixed with 36 kg of rice costing Rs. 5.40 per kg so that 20% gain may be obtained by selling the mixture at Rs. 7.20 per kg?

Sol. S.P. of 1 kg mixture = Rs. 7.20, Gain = 20%.

\therefore C.P. of 1 kg mixture = Rs. $\left(\frac{100}{120} \times 7.20\right) = \text{Rs. } 6$.

By the rule of alligation, we have :

C.P. of 1 kg wheat of 1st kind
(800 p)

C.P. of 1 kg wheat of 2nd kind
(540 p)

Mean price
(600 p)

60

200

Wheat of 1st kind : Wheat of 2nd kind = 60 : 200 = 3 : 10.

Let x kg of wheat of 1st kind be mixed with 36 kg of wheat of 2nd kind.

Then, $3 : 10 = x : 36$ or $10x = 3 \times 36$ or $x = 10.8$ kg.

Ex. 5. The milk and water in two vessels A and B are in the ratio 4 : 3 and 2 : 3 respectively. In what ratio, the liquids in both the vessels be mixed to obtain a new mixture in vessel C containing half milk and half water ?

Sol. Let the C.P. of milk be Re. 1 per litre.

Milk in 1 litre mixture of A = $\frac{4}{7}$ litre; Milk in 1 litre mixture of B = $\frac{2}{5}$ litre;

Milk in 1 litre mixture of C = $\frac{1}{2}$ litre.

\therefore C.P. of 1 litre mixture in A = Re. $\frac{4}{7}$; C.P. of 1 litre mixture in B = Re. $\frac{2}{5}$.

Mean price = Re. $\frac{1}{2}$.

By the rule of alligation, we have :

C.P. of 1 litre mix. in A

C.P. of 1 litre mix. in B

$\left(\frac{4}{7}\right)$

$\left(\frac{2}{5}\right)$

$\left(\frac{1}{2}\right)$

$\left(\frac{1}{10}\right)$

$\left(\frac{1}{14}\right)$

\therefore Required ratio = $\frac{1}{10} : \frac{1}{14} = 7 : 5$.

21. SIMPLE INTEREST

IMPORTANT FACTS AND FORMULAE

1. **Principal** : The money borrowed or lent out for a certain period is called the *principal* or the *sum*.
2. **Interest** : Extra money paid for using other's money is called *interest*.
3. **Simple Interest (S.I.)** : If the interest on a sum borrowed for a certain period is reckoned uniformly, then it is called *simple interest*.

Let Principal = P, Rate = R% per annum (p.a.) and Time = T years. Then,

$$(i) \text{ S.I.} = \left(\frac{P \times R \times T}{100} \right)$$

$$(ii) \text{ P} = \left(\frac{100 \times \text{S.I.}}{R \times T} \right); \text{ R} = \left(\frac{100 \times \text{S.I.}}{P \times T} \right) \text{ and } \text{T} = \left(\frac{100 \times \text{S.I.}}{P \times R} \right)$$

SOLVED EXAMPLES

Ex. 1. Find the simple interest on Rs. 68,000 at $16\frac{2}{3}\%$ per annum for 9 months.

Sol. P = Rs. 68000, R = $\frac{50}{3}\%$ p.a and T = $\frac{9}{12}$ years = $\frac{3}{4}$ years.

$$\therefore \text{ S.I.} = \left(\frac{P \times R \times T}{100} \right) = \text{Rs.} \left(68000 \times \frac{50}{3} \times \frac{3}{4} \times \frac{1}{100} \right) = \text{Rs. 8500.}$$

Ex. 2. Find the simple interest on Rs. 3000 at $6\frac{1}{4}\%$ per annum for the period from 4th Feb., 2005 to 18th April, 2005.

Sol. Time = (24 + 31 + 18) days = 73 days = $\frac{73}{365}$ year = $\frac{1}{5}$ year.

P = Rs. 3000 and R = $6\frac{1}{4}\%$ p.a. = $\frac{25}{4}\%$ p.a.

$$\therefore \text{ S.I.} = \text{Rs.} \left(3000 \times \frac{25}{4} \times \frac{1}{5} \times \frac{1}{100} \right) = \text{Rs. 37.50.}$$

Remark : The day on which money is deposited is not counted while the day on which money is withdrawn is counted.

Ex. 3. A sum at simple interest at $13\frac{1}{2}\%$ per annum amounts to Rs. 2502.50 after 4 years. Find the sum.

Sol. Let sum be Rs. x. Then, S.I. = Rs. $\left(x \times \frac{27}{2} \times 4 \times \frac{1}{100} \right) = \text{Rs.} \frac{27x}{50}$.

$$\therefore \text{ Amount} = \text{Rs.} \left(x + \frac{27x}{50} \right) = \text{Rs.} \frac{77x}{50}.$$

$$\therefore \frac{77x}{50} = 250250 \Leftrightarrow x = \frac{250250 \times 50}{77} = 1625.$$

Hence, sum = Rs. 1625.

Ex. 4. A sum of Rs. 800 amounts to Rs. 920 in 3 years at simple interest. If the interest rate is increased by 3%, it would amount to how much?

Sol. S.I. = Rs. (920 - 800) = Rs. 120; P = Rs. 800, T = 3 yrs.

$$\therefore R = \left(\frac{100 \times 120}{800 \times 3} \right) \% = 5\%.$$

New rate = (5 + 3)% = 8%.

$$\text{New S.I.} = \text{Rs.} \left(\frac{800 \times 8 \times 3}{100} \right) = \text{Rs.} 192.$$

\therefore New amount = Rs. (800 + 192) = Rs. 992.

Ex. 5. Adam borrowed some money at the rate of 6% p.a. for the first two years, at the rate of 9% p.a. for the next three years, and at the rate of 14% p.a. for the period beyond five years. If he pays a total interest of Rs. 11,400 at the end of nine years, how much money did he borrow? (Bank P.O. 1999)

Sol. Let the sum borrowed be x. Then,

$$\left(\frac{x \times 6 \times 2}{100} \right) + \left(\frac{x \times 9 \times 3}{100} \right) + \left(\frac{x \times 14 \times 4}{100} \right) = 11400$$

$$\Leftrightarrow \left(\frac{3x}{25} + \frac{27x}{100} + \frac{14x}{25} \right) = 11400 \Leftrightarrow \frac{95x}{100} = 11400 \Leftrightarrow x = \left(\frac{11400 \times 100}{95} \right) = 12000.$$

Hence, sum borrowed = Rs. 12,000.

Ex. 6. A certain sum of money amounts to Rs. 1008 in 2 years and to Rs. 1164 in $3\frac{1}{2}$ years. Find the sum and the rate of interest.

Sol. S.I. for $1\frac{1}{2}$ years = Rs. (1164 - 1008) = Rs. 156.

$$\text{S.I. for 2 years} = \text{Rs.} \left(156 \times \frac{2}{3} \times 2 \right) = \text{Rs.} 208.$$

\therefore Principal = Rs. (1008 - 208) = Rs. 800.

Now, P = 800, T = 2 and S.I. = 208.

$$\therefore \text{Rate} = \left(\frac{100 \times 208}{800 \times 2} \right) \% = 13\%.$$

Ex. 7. At what rate percent per annum will a sum of money double in 16 years? (R.R.B. 2003)

Sol. Let principal = P. Then, S.I. = P and T = 16 yrs.

$$\therefore \text{Rate} = \left(\frac{100 \times P}{P \times 16} \right) \% = 6\frac{1}{4} \% \text{ p.a.}$$

Ex. 8. The simple interest on a sum of money is $\frac{4}{9}$ of the principal. Find the rate percent and time, if both are numerically equal. (S.S.C. 2000)

Sol. Let sum = Rs. x. Then, S.I. = Rs. $\frac{4x}{9}$.

Let rate = R% and time = R years.

$$\text{Then, } \left(\frac{x \times R \times R}{100} \right) = \frac{4x}{9} \text{ or } R^2 = \frac{400}{9} \text{ or } R = \frac{20}{3} = 6\frac{2}{3}.$$

$$\therefore \text{Rate} = 6\frac{2}{3}\% \text{ and Time} = 6\frac{2}{3} \text{ yrs} = 6 \text{ yrs } 8 \text{ months.}$$

Ex. 9. The simple interest on a certain sum of money for $2\frac{1}{2}$ years at 12% per annum is Rs. 40 less than the simple interest on the same sum for $3\frac{1}{2}$ years at 10% per annum. Find the sum.

$$\text{Sol. Let the sum be Rs. } x \text{ Then, } \left(\frac{x \times 10 \times 7}{100 \times 2} \right) - \left(\frac{x \times 12 \times 5}{100 \times 2} \right) = 40$$

$$\Leftrightarrow \frac{7x}{20} - \frac{3x}{10} = 40 \Leftrightarrow x = (40 \times 20) = 800.$$

Hence, the sum is Rs. 800.

Ex. 10. A sum was put at simple interest at a certain rate for 3 years. Had it been put at 2% higher rate, it would have fetched Rs. 360 more. Find the sum.

$$\text{Sol. Let sum} = P \text{ and original rate} = R. \text{ Then, } \left[\frac{P \times (R + 2) \times 3}{100} \right] - \left[\frac{P \times R \times 3}{100} \right] = 360$$

$$\Leftrightarrow 3PR + 6P - 3PR = 36000 \Leftrightarrow 6P = 36000 \Leftrightarrow P = 6000.$$

Hence, sum = Rs. 6000.

Ex. 11. What annual instalment will discharge a debt of Rs. 1092 due in 3 years at 12% simple interest?

$$\text{Sol. Let each instalment be Rs. } x \text{ Then, } \left(x + \frac{x \times 12 \times 1}{100} \right) + \left(x + \frac{x \times 12 \times 2}{100} \right) + x = 1092$$

$$\Leftrightarrow \frac{28x}{25} + \frac{31x}{25} + x = 1092 \Leftrightarrow (28x + 31x + 25x) = (1092 \times 25)$$

$$\Leftrightarrow x = \left(\frac{1092 \times 25}{84} \right) = 325.$$

\therefore Each instalment = Rs. 325.

Ex. 12. A sum of Rs. 1550 is lent out into two parts, one at 8% and another one at 6%. If the total annual income is Rs. 106, find the money lent at each rate.

(L.I.C. A.A.O. 2003)

Sol. Let the sum lent at 8% be Rs. x and that at 6% be Rs. $(1550 - x)$.

$$\therefore \left[\frac{x \times 8 \times 1}{100} \right] + \left[\frac{(1550 - x) \times 6 \times 1}{100} \right] = 106$$

$$\Leftrightarrow 8x + 9300 - 6x = 10600 \Leftrightarrow 2x = 1300 \Leftrightarrow x = 650.$$

\therefore Money lent at 8% = Rs. 650. Money lent at 6% = Rs. $(1550 - 650) = \text{Rs. } 900.$

22. COMPOUND INTEREST

Compound Interest : Sometimes it so happens that the borrower and the lender agree to fix up a certain unit of time, say *yearly* or *half-yearly* or *quarterly* to settle the previous account.

In such cases, the amount after first unit of time becomes the principal for the second unit, the amount after second unit becomes the principal for the third unit and so on.

After a specified period, *the difference between the amount and the money borrowed is called the Compound Interest (abbreviated as C.I.) for that period.*

IMPORTANT FACTS AND FORMULAE

Let Principal = P, Rate = R% per annum, Time = n years.

I. When interest is compound Annually :

$$\text{Amount} = P \left(1 + \frac{R}{100} \right)^n$$

II. When interest is compounded Half-yearly :

$$\text{Amount} = P \left[1 + \frac{(R/2)}{100} \right]^{2n}$$

III. When interest is compounded Quarterly :

$$\text{Amount} = P \left[1 + \frac{(R/4)}{100} \right]^{4n}$$

IV. When interest is compounded Annually but time is in fraction, say $3\frac{2}{5}$ years.

$$\text{Amount} = P \left(1 + \frac{R}{100} \right)^3 \times \left(1 + \frac{\frac{2}{5}R}{100} \right)$$

V. When Rates are different for different years, say $R_1\%$, $R_2\%$, $R_3\%$ for 1st, 2nd and 3rd year respectively.

$$\text{Then, Amount} = P \left(1 + \frac{R_1}{100} \right) \left(1 + \frac{R_2}{100} \right) \left(1 + \frac{R_3}{100} \right).$$

VI. Present worth of Rs. x due n years hence is given by :

$$\text{Present Worth} = \frac{x}{\left(1 + \frac{R}{100} \right)^n}.$$

SOLVED EXAMPLES

Ex. 1. Find compound interest on Rs. 7500 at 4% per annum for 2 years, compounded annually.

$$\text{Sol. Amount} = \text{Rs. } \left[7500 \times \left(1 + \frac{4}{100} \right)^2 \right] = \text{Rs. } \left(7500 \times \frac{26}{25} \times \frac{26}{25} \right) = \text{Rs. 8112.}$$

$$\therefore \text{C.I.} = \text{Rs. } (8112 - 7500) = \text{Rs. 612.}$$

Ex. 2. Find compound interest on Rs. 8000 at 15% per annum for 2 years 4 months, compounded annually.

$$\text{Sol. Time} = 2 \text{ years } 4 \text{ months} = 2\frac{4}{12} \text{ years} = 2\frac{1}{3} \text{ years.}$$

$$\begin{aligned} \text{Amount} &= \text{Rs. } \left[8000 \times \left(1 + \frac{15}{100} \right)^2 \times \left(1 + \frac{\frac{1}{3} \times 15}{100} \right) \right] = \text{Rs. } \left(8000 \times \frac{23}{20} \times \frac{23}{20} \times \frac{21}{20} \right) \\ &= \text{Rs. 11109.} \end{aligned}$$

$$\therefore \text{C.I.} = \text{Rs. } (11109 - 8000) = \text{Rs. 3109.}$$

Ex. 3. Find the compound interest on Rs. 10,000 in 2 years at 4% per annum, the interest being compounded half-yearly. (S.S.C. 2000)

Sol. Principal = Rs. 10000; Rate = 2% per half-year; Time = 2 years = 4 half-years.

$$\begin{aligned} \therefore \text{Amount} &= \text{Rs. } \left[10000 \times \left(1 + \frac{2}{100} \right)^4 \right] = \text{Rs. } \left(10000 \times \frac{51}{50} \times \frac{51}{50} \times \frac{51}{50} \times \frac{51}{50} \right) \\ &= \text{Rs. 10824.32.} \end{aligned}$$

$$\therefore \text{C.I.} = \text{Rs. } (10824.32 - 10000) = \text{Rs. 824.32.}$$

Ex. 4. Find the compound interest on Rs. 16,000 at 20% per annum for 9 months, compounded quarterly.

Sol. Principal = Rs. 16000; Time = 9 months = 3 quarters;

Rate = 20% per annum = 5% per quarter.

$$\therefore \text{Amount} = \text{Rs. } \left[16000 \times \left(1 + \frac{5}{100} \right)^3 \right] = \text{Rs. } \left(16000 \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20} \right) = \text{Rs. 18522.}$$

$$\therefore \text{C.I.} = \text{Rs. } (18522 - 16000) = \text{Rs. 2522.}$$

Ex. 5. If the simple interest on a sum of money at 5% per annum for 3 years is Rs. 1200, find the compound interest on the same sum for the same period at the same rate.

Sol. Clearly, Rate = 5% p.a., Time = 3 years, S.I. = Rs. 1200.

$$\text{So, Principal} = \text{Rs. } \left(\frac{100 \times 1200}{3 \times 5} \right) = \text{Rs. 8000.}$$

$$\text{Amount} = \text{Rs. } \left[8000 \times \left(1 + \frac{5}{100} \right)^3 \right] = \text{Rs. } \left(8000 \times \frac{21}{20} \times \frac{21}{20} \times \frac{21}{20} \right) = \text{Rs. 9261.}$$

$$\therefore \text{C.I.} = \text{Rs. } (9261 - 8000) = \text{Rs. 1261.}$$

Ex. 6. In what time will Rs. 1900 become Rs. 1331 at 10% per annum compounded annually? (S.S.C. 2004)

Sol. Principal = Rs. 1000; Amount = Rs. 1331; Rate = 10% p.a.

Let the time be n years. Then,

$$\left[1000 \left(1 + \frac{10}{100} \right)^n \right] = 1331 \text{ or } \left(\frac{11}{10} \right)^n = \left(\frac{1331}{1000} \right) = \left(\frac{11}{10} \right)^3$$

$\therefore n = 3$ years.

Ex. 7. If Rs. 500 amounts to Rs. 583.20 in two years compounded annually, find the rate of interest per annum.

Sol. Principal = Rs. 500; Amount = Rs. 583.20; Time = 2 years.

Let the rate be $R\%$ per annum. Then,

$$\left[500 \left(1 + \frac{R}{100} \right)^2 \right] = 583.20 \text{ or } \left(1 + \frac{R}{100} \right)^2 = \frac{5832}{5000} = \frac{11664}{10000}$$

$$\therefore \left(1 + \frac{R}{100} \right)^2 = \left(\frac{108}{100} \right)^2 \text{ or } 1 + \frac{R}{100} = \frac{108}{100} \text{ or } R = 8.$$

So, rate = 8% p.a.

Ex. 8. If the compound interest on a certain sum at $16\frac{2}{3}\%$ for 3 years is Rs. 1270, find the simple interest on the same sum at the same rate and for the same period.

Sol. Let the sum be Rs. x . Then,

$$\text{C.I.} = \left[x \times \left(1 + \frac{50}{3 \times 100} \right)^3 - x \right] = \left(\frac{343x}{216} - x \right) = \frac{127x}{216}$$

$$\therefore \frac{127x}{216} = 1270 \text{ or } x = \frac{1270 \times 216}{127} = 2160.$$

Thus, the sum is Rs. 2160.

$$\therefore \text{S.I.} = \text{Rs.} \left(2160 \times \frac{50}{3} \times 3 \times \frac{1}{100} \right) = \text{Rs. } 1080.$$

Ex. 9. The difference between the compound interest and simple interest on a certain sum at 10% per annum for 2 years is Rs. 631. Find the sum.

Sol. Let the sum be Rs. x . Then,

$$\text{C.I.} = x \left(1 + \frac{10}{100} \right)^2 - x = \frac{21x}{100}, \text{ S.I.} = \left(\frac{x \times 10 \times 2}{100} \right) = \frac{x}{5}$$

$$\therefore (\text{C.I.}) - (\text{S.I.}) = \left(\frac{21x}{100} - \frac{x}{5} \right) = \frac{x}{100}$$

$$\therefore \frac{x}{100} = 631 \Rightarrow x = 63100.$$

Hence, the sum is Rs. 63,100.

Ex. 10. The difference between the compound interest and the simple interest accrued on an amount of Rs. 18,000 in 2 years was Rs. 405. What was the rate of interest p.c.p.a. ? (Bank P.O. 2003)

Sol. Let the rate be $R\%$ p.a. Then,

$$\left[18000 \left(1 + \frac{R}{100} \right)^2 - 18000 \right] - \left(\frac{18000 \times R \times 2}{100} \right) = 405$$

$$\Rightarrow 18000 \left[\frac{(100 + R)^2}{10000} - 1 - \frac{2R}{100} \right] = 405$$

$$\Leftrightarrow 18000 \left[\frac{(100 + R)^2 - 10000 - 200R}{10000} \right] = 405$$

$$\Leftrightarrow \frac{9}{5} R^2 = 405 \Leftrightarrow R^2 = \left(\frac{405 \times 5}{9} \right) = 225 \Leftrightarrow R = 15.$$

\therefore Rate = 15%.

Ex. 11. Divide Rs. 1301 between A and B, so that the amount of A after 7 years is equal to the amount of B after 9 years, the interest being compounded at 4% per annum.

Sol. Let the two parts be Rs. x and Rs. $(1301 - x)$.

$$x \left(1 + \frac{4}{100} \right)^7 = (1301 - x) \left(1 + \frac{4}{100} \right)^9 \Leftrightarrow \frac{x}{(1301 - x)} = \left(1 + \frac{4}{100} \right)^2 = \left(\frac{26}{25} \times \frac{26}{25} \right).$$

$$\Leftrightarrow 625x = 676 (1301 - x) \Leftrightarrow 1301x = 676 \times 1301 \Leftrightarrow x = 676.$$

So, the two parts are Rs. 676 and Rs. $(1301 - 676)$ i.e. Rs. 676 and Rs. 625.

Ex. 12. A certain sum amounts to Rs. 7350 in 2 years and to Rs. 8575 in 3 years. Find the sum and rate percent.

Sol. S.I. on Rs. 7350 for 1 year = Rs. $(8575 - 7350)$ = Rs. 1225.

$$\therefore \text{Rate} = \left(\frac{100 \times 1225}{7350 \times 1} \right) \% = 16\frac{2}{3}\%.$$

Let the sum be Rs. x . Then,

$$x \left(1 + \frac{50}{3 \times 100} \right)^2 = 7350 \Leftrightarrow x \times \frac{7}{6} \times \frac{7}{6} = 7350 \Leftrightarrow x = \left(7350 \times \frac{36}{49} \right) = 5400.$$

\therefore Sum = Rs. 5400.

Ex. 13. A sum of money amounts to Rs. 6690 after 3 years and to Rs. 10,035 after 6 years on compound interest. Find the sum.

Sol. Let the sum be Rs. P . Then,

$$P \left(1 + \frac{R}{100} \right)^3 = 6690 \quad \dots (i) \quad \text{and} \quad P \left(1 + \frac{R}{100} \right)^6 = 10035 \quad \dots (ii)$$

$$\text{On dividing, we get } \left(1 + \frac{R}{100} \right)^3 = \frac{10035}{6690} = \frac{3}{2}.$$

Substituting this value in (i), we get :

$$P \times \frac{3}{2} = 6690 \text{ or } P = \left(6690 \times \frac{2}{3} \right) = 4460.$$

Hence, the sum is Rs. 4460.

Ex. 14. A sum of money doubles itself at compound interest in 15 years. In how many years will it become eight times ?

$$\text{Sol. } P \left(1 + \frac{R}{100} \right)^{15} = 2P \Rightarrow \left(1 + \frac{R}{100} \right)^{15} = \frac{2P}{P} = 2 \quad \dots (i)$$

$$\text{Let } P \left(1 + \frac{R}{100} \right)^n = 8P \Rightarrow \left(1 + \frac{R}{100} \right)^n = 8 = 2^3 = \left\{ \left(1 + \frac{R}{100} \right)^{15} \right\}^3 \quad [\text{using (i)}]$$

$$\Rightarrow \left(1 + \frac{R}{100} \right)^n = \left(1 + \frac{R}{100} \right)^{45} \Rightarrow n = 45.$$

Thus, the required time = 45 years.

Ex. 15. What annual payment will discharge a debt of Rs. 7620 due in 3 years at $16\frac{2}{3}\%$ per annum compound interest?

Sol. Let each instalment be Rs. x . Then,

$$\begin{aligned} &(\text{P.W. of Rs. } x \text{ due 1 year hence}) + (\text{P.W. of Rs. } x \text{ due 2 years hence}) \\ &+ (\text{P.W. of Rs. } x \text{ due 3 years hence}) = 7620. \end{aligned}$$

$$\therefore \frac{x}{\left(1 + \frac{50}{3 \times 100}\right)} + \frac{x}{\left(1 + \frac{50}{3 \times 100}\right)^2} + \frac{x}{\left(1 + \frac{50}{3 \times 100}\right)^3} = 7620$$

$$\Leftrightarrow \frac{6x}{7} + \frac{36x}{49} + \frac{216x}{343} = 7620 \Leftrightarrow 294x + 252x + 216x = 7620 \times 343$$

$$\Leftrightarrow x = \left(\frac{7620 \times 343}{762}\right) = 3430.$$

\therefore Amount of each instalment = Rs. 3430.

23. LOGARITHMS

IMPORTANT FACTS AND FORMULAE

I. Logarithm : If a is a positive real number, other than 1 and $a^m = x$, then we write : $m = \log_a x$ and we say that the value of $\log x$ to the base a is m .

Example :

$$\begin{aligned} \text{(i)} \quad 10^3 &= 1000 \Rightarrow \log_{10} 1000 = 3 & \text{(ii)} \quad 3^4 &= 81 \Rightarrow \log_3 81 = 4 \\ \text{(ii)} \quad 2^{-3} &= \frac{1}{8} \Rightarrow \log_2 \frac{1}{8} = -3 & \text{(iv)} \quad (.1)^2 &= .01 \Rightarrow \log_{(.1)} .01 = 2. \end{aligned}$$

II. Properties of Logarithms :

$$\begin{aligned} 1. \log_a (xy) &= \log_a x + \log_a y & 2. \log_a \left(\frac{x}{y} \right) &= \log_a x - \log_a y \\ 3. \log_x x &= 1 & 4. \log_a 1 &= 0 \\ 5. \log_a (x^p) &= p (\log_a x) & 6. \log_a x &= \frac{1}{\log_x a} \\ 7. \log_a x &= \frac{\log_b x}{\log_b a} = \frac{\log x}{\log a} \end{aligned}$$

Remember : When base is not mentioned, it is taken as 10.

III. Common Logarithms : Logarithms to the base 10 are known as common logarithms.

IV. The logarithm of a number contains two parts, namely *characteristic* and *mantissa*.

Characteristic : The integral part of the logarithm of a number is called its *characteristic*.

Case I : When the number is greater than 1.

In this case, the characteristic is one less than the number of digits in the left of the decimal point in the given number.

Case II : When the number is less than 1.

In this case, the characteristic is one more than the number of zeros between the decimal point and the first significant digit of the number and it is negative.

Instead of -1 , -2 , etc. we write, $\bar{1}$ (one bar), $\bar{2}$ (two bar), etc.

Example :

Number	Characteristic	Number	Characteristic
348.25	2	0.6173	$\bar{1}$
46.583	1	0.03125	$\bar{2}$
9.2193	0	0.00125	$\bar{3}$

Mantissa : The decimal part of the logarithm of a number is known as its *mantissa*. For mantissa, we look through log table.

SOLVED EXAMPLES

Ex. 1. Evaluate : (i) $\log_3 27$ (ii) $\log_7 \left(\frac{1}{343} \right)$ (iii) $\log_{100} (0.01)$

Sol. (i) Let $\log_3 27 = n$.
Then, $3^n = 27 = 3^3$ or $n = 3$. $\therefore \log_3 27 = 3$.

(ii) Let $\log_7 \left(\frac{1}{343} \right) = n$.
Then, $7^n = \frac{1}{343} = \frac{1}{7^3} = 7^{-3}$ or $n = -3$. $\therefore \log_7 \left(\frac{1}{343} \right) = -3$.

(iii) Let $\log_{100} (0.01) = n$.
Then, $(100)^n = 0.01 = \frac{1}{100} = (100)^{-1}$ or $n = -1$. $\therefore \log_{100} (0.01) = -1$.

Ex. 2. Evaluate : (i) $\log_7 1 = 0$ (ii) $\log_{34} 34$ (iii) $36^{\log_6 4}$

Sol. (i) We know that $\log_a 1 = 0$, so $\log_7 1 = 0$.

(ii) We know that $\log_a a = 1$, so $\log_{34} 34 = 1$.

(iii) We know that $a^{\log_a x} = x$.

Now, $36^{\log_6 4} = (6^2)^{\log_6 4} = 6^{2(\log_6 4)} = 6^{\log_6 (4^2)} = 6^{\log_6 16} = 16$.

Ex. 3. If $\log_{\sqrt{8}} x = 3\frac{1}{3}$, find the value of x .

Sol. $\log_{\sqrt{8}} x = \frac{10}{3} \Leftrightarrow x = (\sqrt{8})^{10/3} = (2^{3/2})^{10/3} = 2^{\left(\frac{3}{2} \times \frac{10}{3}\right)} = 2^5 = 32$.

Ex. 4. Evaluate : (i) $\log_5 3 \times \log_{27} 25$ (ii) $\log_9 27 - \log_{27} 9$

Sol. (i) $\log_5 3 \times \log_{27} 25 = \frac{\log 3}{\log 5} \times \frac{\log 25}{\log 27} = \frac{\log 3}{\log 5} \times \frac{\log (5^2)}{\log (3^3)} = \frac{\log 3}{\log 5} \times \frac{2 \log 5}{3 \log 3} = \frac{2}{3}$.

(ii) Let $\log_9 27 = n$.

Then, $9^n = 27 \Leftrightarrow 3^{2n} = 3^3 \Leftrightarrow 2n = 3 \Leftrightarrow n = \frac{3}{2}$.

Again, let $\log_{27} 9 = m$.

Then, $27^m = 9 \Leftrightarrow 3^{3m} = 3^2 \Leftrightarrow 3m = 2 \Leftrightarrow m = \frac{2}{3}$.

$\therefore \log_9 27 - \log_{27} 9 = (n - m) = \left(\frac{3}{2} - \frac{2}{3} \right) = \frac{5}{6}$.

Ex. 5. Simplify : $\left(\log \frac{75}{16} - 2 \log \frac{5}{9} + \log \frac{32}{243} \right)$ (S.S.C. 2000)

Sol. $\log \frac{75}{16} - 2 \log \frac{5}{9} + \log \frac{32}{243} = \log \frac{75}{16} - \log \left(\frac{5}{9} \right)^2 + \log \frac{32}{243} = \log \frac{75}{16} - \log \frac{25}{81} + \log \frac{32}{243}$
 $= \log \left(\frac{75}{16} \times \frac{81}{25} \times \frac{32}{243} \right) = \log 2$

Ex. 6. Find the value of x which satisfies the relation

$$\log_{10} 3 + \log_{10} (4x + 1) = \log_{10} (x + 1) + 1 \quad (\text{M.B.A. 2002})$$

$$\text{Sol. } \log_{10} 3 + \log_{10} (4x + 1) = \log_{10} (x + 1) + 1$$

$$\Leftrightarrow \log_{10} 3 + \log_{10} (4x + 1) = \log_{10} (x + 1) + \log_{10} 10$$

$$\Leftrightarrow \log_{10} [3 (4x + 1)] = \log_{10} [10 (x + 1)]$$

$$\Leftrightarrow 3 (4x + 1) = 10 (x + 1) \Leftrightarrow 12x + 3 = 10x + 10 \Leftrightarrow 2x = 7 \Leftrightarrow x = \frac{7}{2}$$

$$\text{Ex. 7. Simplify : } \left[\frac{1}{\log_{xy} (xyz)} + \frac{1}{\log_{yz} (xyz)} + \frac{1}{\log_{zx} (xyz)} \right]$$

$$\text{Sol. Given expression} = \log_{xyz} (xy) + \log_{xyz} (yz) + \log_{xyz} (zx)$$

$$= \log_{xyz} (xy \times yz \times zx) = \log_{xyz} (xyz)^2$$

$$= 2 \log_{xyz} (xyz) = 2 \times 1 = 2.$$

$$\left[\because \log_a x = \frac{1}{\log_x a} \right]$$

Ex. 8. If $\log_{10} 2 = 0.30103$, find the value of $\log_{10} 50$.

(C.B.I. 1997)

$$\text{Sol. } \log_{10} 50 = \log_{10} \left(\frac{100}{2} \right) = \log_{10} 100 - \log_{10} 2 = 2 - 0.30103 = 1.69897.$$

Ex. 9. If $\log 2 = 0.3010$ and $\log 3 = 0.4771$, find the values of :

(i) $\log 25$

(ii) $\log 4.5$

$$\text{Sol. (i) } \log 25 = \log \left(\frac{100}{4} \right) = \log 100 - \log 4 = 2 - 2 \log 2 = (2 - 2 \times 0.3010) = 1.398.$$

$$(ii) \log 4.5 = \log \left(\frac{9}{2} \right) = \log 9 - \log 2 = 2 \log 3 - \log 2$$

$$= (2 \times 0.4771 - 0.3010) = 0.6532$$

Ex. 10. If $\log 2 = 0.30103$, find the number of digits in 2^{56} .

$$\text{Sol. } \log (2^{56}) = 56 \log 2 = (56 \times 0.30103) = 16.85768.$$

Its characteristic is 16. Hence, the number of digits in 2^{56} is 17.

24. AREA

FUNDAMENTAL CONCEPTS

I. Results on Triangles :

1. Sum of the angles of a triangle is 180° .
2. The sum of any two sides of a triangle is greater than the third side.
3. **Pythagoras Theorem** : In a right-angled triangle,
 $(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Height})^2$.
4. The line joining the mid-point of a side of a triangle to the opposite vertex is called the **median**.
5. The point where the three medians of a triangle meet, is called **centroid**. The centroid divides each of the medians in the ratio 2 : 1.
6. In an isosceles triangle, the altitude from the vertex bisects the base.
7. The median of a triangle divides it into two triangles of the same area.
8. The area of the triangle formed by joining the mid-points of the sides of a given triangle is one-fourth of the area of the given triangle.

II. Results on Quadrilaterals :

1. The diagonals of a parallelogram bisect each other.
2. Each diagonal of a parallelogram divides it into two triangles of the same area.
3. The diagonals of a rectangle are equal and bisect each other.
4. The diagonals of a square are equal and bisect each other at right angles.
5. The diagonals of a rhombus are unequal and bisect each other at right angles.
6. A parallelogram and a rectangle on the same base and between the same parallels are equal in area.
7. Of all the parallelogram of given sides, the parallelogram which is a rectangle has the greatest area.

IMPORTANT FORMULAE

- I. 1. Area of a rectangle = (Length \times Breadth).

$$\therefore \text{Length} = \left(\frac{\text{Area}}{\text{Breadth}} \right) \text{ and } \text{Breadth} = \left(\frac{\text{Area}}{\text{Length}} \right)$$

2. Perimeter of a rectangle = 2 (Length + Breadth).

II. Area of a square = $(\text{side})^2 = \frac{1}{2} (\text{diagonal})^2$.

III. Area of 4 walls of a room = 2 (Length + Breadth) \times Height.

IV. 1. Area of a triangle = $\frac{1}{2} \times \text{Base} \times \text{Height}$.

2. Area of a triangle = $\sqrt{s(s-a)(s-b)(s-c)}$, where a, b, c are the sides of the triangle and $s = \frac{1}{2}(a+b+c)$.

$$3. \text{ Area of an equilateral triangle} = \frac{\sqrt{3}}{4} \times (\text{side})^2.$$

$$4. \text{ Radius of incircle of an equilateral triangle of side } a = \frac{a}{2\sqrt{3}}.$$

$$5. \text{ Radius of circumcircle of an equilateral triangle of side } a = \frac{a}{\sqrt{3}}.$$

$$6. \text{ Radius of incircle of a triangle of area } \Delta \text{ and semi-perimeter } s = \frac{\Delta}{s}.$$

V. 1. Area of a parallelogram = (Base \times Height).

$$2. \text{ Area of a rhombus} = \frac{1}{2} \times (\text{Product of diagonals}).$$

$$3. \text{ Area of a trapezium} = \frac{1}{2} \times (\text{sum of parallel sides}) \times \text{distance between them}.$$

VI. 1. Area of a circle = πR^2 , where R is the radius.

$$2. \text{ Circumference of a circle} = 2\pi R.$$

$$3. \text{ Length of an arc} = \frac{2\pi R\theta}{360}, \text{ where } \theta \text{ is the central angle.}$$

$$4. \text{ Area of a sector} = \frac{1}{2} (\text{arc} \times R) = \frac{\pi R^2 \theta}{360}.$$

$$\text{VII. 1. Area of a semi-circle} = \frac{\pi R^2}{2}.$$

$$2. \text{ Circumference of a semi-circle} = \pi R.$$

SOLVED EXAMPLES

Ex. 1. One side of a rectangular field is 15 m and one of its diagonals is 17 m. Find the area of the field.

$$\text{Sol. Other side} = \sqrt{(17)^2 - (15)^2} = \sqrt{289 - 225} = \sqrt{64} = 8 \text{ m.}$$

$$\therefore \text{ Area} = (15 \times 8) \text{ m}^2 = 120 \text{ m}^2.$$

Ex. 2. A lawn is in the form of a rectangle having its sides in the ratio 2 : 3. The area of the lawn is $\frac{1}{6}$ hectares. Find the length and breadth of the lawn.

$$\text{Sol. Let length} = 2x \text{ metres and breadth} = 3x \text{ metres.}$$

$$\text{Now, area} = \left(\frac{1}{6} \times 1000\right) \text{ m}^2 = \left(\frac{5000}{3}\right) \text{ m}^2.$$

$$\text{So, } 2x \times 3x = \frac{5000}{3} \Leftrightarrow x^2 = \frac{2500}{9} \Leftrightarrow x = \left(\frac{50}{3}\right).$$

$$\therefore \text{ Length} = 2x = \frac{100}{3} \text{ m} = 33\frac{1}{3} \text{ m and Breadth} = 3x = \left(3 \times \frac{50}{3}\right) \text{ m} = 50 \text{ m.}$$

Ex. 3. Find the cost of carpeting a room 13 m long and 9 m broad with a carpet 75 cm wide at the rate of Rs. 12.40 per square metre.

$$\text{Sol. Area of the carpet} = \text{Area of the room} = (13 \times 9) \text{ m}^2 = 117 \text{ m}^2.$$

$$\text{Length of the carpet} = \left(\frac{\text{Area}}{\text{Width}}\right) = \left(117 \times \frac{4}{3}\right) \text{ m} = 156 \text{ m.}$$

$$\therefore \text{ Cost of carpeting} = \text{Rs. } (156 \times 12.40) = \text{Rs. } 1934.40.$$

Ex. 4. If the diagonal of a rectangle is 17 cm long and its perimeter is 46 cm, find the area of the rectangle.

Sol. Let length = x and breadth = y . Then,

$$2(x + y) = 46 \text{ or } x + y = 23 \text{ and } x^2 + y^2 = (17)^2 = 289.$$

$$\text{Now, } (x + y)^2 = (23)^2 \Rightarrow (x^2 + y^2) + 2xy = 529 \Rightarrow 289 + 2xy = 529 \Rightarrow xy = 120$$

$$\text{Area} = xy = 120 \text{ cm}^2$$

Ex. 5. The length of a rectangle is twice its breadth. If its length is decreased by 5 cm and breadth is increased by 5 cm, the area of the rectangle is increased by 75 sq. cm. Find the length of the rectangle.

Sol. Let breadth = x . Then, length = $2x$. Then,

$$(2x - 5)(x + 5) - 2x \cdot x = 75 \Rightarrow 5x - 25 = 75 \Rightarrow x = 20.$$

$$\therefore \text{Length of the rectangle} = 20 \text{ cm.}$$

Ex. 6. In measuring the sides of a rectangle, one side is taken 5% in excess, and the other 4% in deficit. Find the error percent in the area calculated from these measurements. (M.B.A. 2003)

Sol. Let x and y be the sides of the rectangle. Then, Correct area = xy .

$$\text{Calculated area} = (105/100) \cdot x \cdot (96/100) \cdot y = (504/500)(xy)$$

$$\text{Error In measurement} = (504/500)xy - xy = (4/500)xy$$

$$\text{Error \%} = [(4/500)xy \cdot (1/xy) \cdot 100] \% = (4/5) \% = 0.8\%.$$

Ex. 7. A rectangular grassy plot 110 m. by 65 m has a gravel path 2.5 m wide all round it on the inside. Find the cost of gravelling the path at 80 paise per sq. metre.

$$\text{Sol. Area of the plot} = (110 \times 65) \text{ m}^2 = 7150 \text{ m}^2$$

$$\text{Area of the plot excluding the path} = [(110 - 5) \cdot (65 - 5)] \text{ m}^2 = 6300 \text{ m}^2.$$

$$\text{Area of the path} = (7150 - 6300) \text{ m}^2 = 850 \text{ m}^2.$$

$$\text{Cost of gravelling the path} = \text{Rs. } 850 \cdot (80/100) = \text{Rs. } 680$$

Ex. 8. The perimeters of two squares are 40 cm and 32 cm. Find the perimeter of a third square whose area is equal to the difference of the areas of the two squares. (S.S.C. 2003)

$$\text{Sol. Side of first square} = (40/4) = 10 \text{ cm;}$$

$$\text{Side of second square} = (32/4) \text{ cm} = 8 \text{ cm.}$$

$$\text{Area of third square} = [(10)^2 - (8)^2] \text{ cm}^2 = (100 - 64) \text{ cm}^2 = 36 \text{ cm}^2.$$

$$\text{Side of third square} = (36)^{(1/2)} \text{ cm} = 6 \text{ cm.}$$

$$\text{Required perimeter} = (6 \times 4) \text{ cm} = 24 \text{ cm.}$$

25. VOLUME AND SURFACE AREA

IMPORTANT FORMULAE

I. CUBOID

Let length = l , breadth = b and height = h units. Then,

1. **Volume** = $(l \times b \times h)$ cubic units.
2. **Surface area** = $2(lb + bh + lh)$ sq. units.
3. **Diagonal** = $\sqrt{l^2 + b^2 + h^2}$ units.

II. CUBE

Let each edge of a cube be of length a . Then,

1. **Volume** = a^3 cubic units.
2. **Surface area** = $6a^2$ sq. units.
3. **Diagonal** = $\sqrt{3}a$ units.

III. CYLINDER

Let radius of base = r and Height (or length) = h . Then,

1. **Volume** = $(\pi r^2 h)$ cubic units.
2. **Curved surface area** = $(2\pi rh)$ sq. units.
3. **Total surface area** = $(2\pi rh + 2\pi r^2)$ sq. units
= $2\pi r(h + r)$ sq. units.

IV. CONE

Let radius of base = r and Height = h . Then,

1. **Slant height**, $l = \sqrt{h^2 + r^2}$ units.
2. **Volume** = $\left(\frac{1}{3} \pi r^2 h\right)$ cubic units.
3. **Curved surface area** = (πrl) sq. units.
4. **Total surface area** = $(\pi rl + \pi r^2)$ sq. units.

V. SPHERE

Let the radius of the sphere be r . Then,

1. **Volume** = $\left(\frac{4}{3} \pi r^3\right)$ cubic units.
2. **Surface area** = $(4\pi r^2)$ sq. units.

VI. HEMISPHERE

Let the radius of a hemisphere be r . Then,

1. **Volume** = $\left(\frac{2}{3} \pi r^3\right)$ cubic units.
2. **Curved surface area** = $(2\pi r^2)$ sq. units.
3. **Total surface area** = $(3\pi r^2)$ sq. units.

Remember : 1 litre = 1000 cm³.

SOLVED EXAMPLES

Ex. 1. Find the volume and surface area of a cuboid 16 m long, 14 m broad and 7 m high.

Sol. Volume = $(16 \times 14 \times 7) \text{ m}^3 = 1568 \text{ m}^3$.

Surface area = $[2(16 \times 14 + 14 \times 7 + 16 \times 7)] \text{ cm}^2 = (2 \times 434) \text{ cm}^2 = 868 \text{ cm}^2$.

Ex. 2. Find the length of the longest pole that can be placed in a room 12 m long, 8 m broad and 9 m high.

Sol. Length of longest pole = Length of the diagonal of the room

$$= \sqrt{(12)^2 + 8^2 + 9^2} = \sqrt{289} = 17 \text{ m.}$$

Ex. 3. The volume of a wall, 5 times as high as it is broad and 8 times as long as it is high, is 12.8 cu. metres. Find the breadth of the wall.

Sol. Let the breadth of the wall be x metres.

Then, Height = $5x$ metres and Length = $40x$ metres.

$$\therefore x \times 5x \times 40x = 12.8 \Leftrightarrow x^3 = \frac{12.8}{200} = \frac{128}{2000} = \frac{64}{1000}$$

$$\text{So, } x = \frac{4}{10} \text{ m} = \left(\frac{4}{10} \times 100\right) \text{ cm} = 40 \text{ cm.}$$

Ex. 4. Find the number of bricks, each measuring $24 \text{ cm} \times 12 \text{ cm} \times 8 \text{ cm}$, required to construct a wall 24 m long, 8 m high and 60 cm thick, if 10% of the wall is filled with mortar?

Sol. Volume of the wall = $(2400 \times 800 \times 60) \text{ cu. cm.}$

Volume of bricks = 90% of the volume of the wall

$$= \left(\frac{90}{100} \times 2400 \times 800 \times 60\right) \text{ cu. cm.}$$

Volume of 1 brick = $(24 \times 12 \times 8) \text{ cu. cm.}$

$$\therefore \text{Number of bricks} = \left(\frac{90}{100} \times \frac{2400 \times 800 \times 60}{24 \times 12 \times 8}\right) = 45000.$$

Ex. 5. Water flows into a tank $200 \text{ m} \times 150 \text{ m}$ through a rectangular pipe $1.5 \text{ m} \times 1.25 \text{ m}$ @ 20 kmph . In what time (in minutes) will the water rise by 2 metres?

Sol. Volume required in the tank = $(200 \times 150 \times 2) \text{ m}^3 = 60000 \text{ m}^3$.

Length of water column flown in 1 min. = $\left(\frac{20 \times 1000}{60}\right) \text{ m} = \frac{1000}{3} \text{ m.}$

Volume flown per minute = $\left(1.5 \times 1.25 \times \frac{1000}{3}\right) \text{ m}^3 = 625 \text{ m}^3$.

$$\therefore \text{Required time} = \left(\frac{60000}{625}\right) \text{ min.} = 96 \text{ min.}$$

Ex. 6. The dimensions of an open box are 50 cm, 40 cm and 23 cm. Its thickness is 3 cm. If 1 cubic cm of metal used in the box weighs 0.5 gms, find the weight of the box.

Sol. Volume of the metal used in the box = External Volume - Internal Volume

$$= [(50 \times 40 \times 23) - (44 \times 34 \times 20)] \text{ cm}^3$$

$$= 16080 \text{ cm}^3.$$

$$\therefore \text{Weight of the metal} = \left(\frac{16080 \times 0.5}{1000}\right) \text{ kg} = 8.04 \text{ kg.}$$

Ex. 7. The diagonal of a cube is $6\sqrt{3}$ cm. Find its volume and surface area.

Sol. Let the edge of the cube be a .

$$\therefore \sqrt{3} a = 6\sqrt{3} \Rightarrow a = 6.$$

$$\text{So, Volume} = a^3 = (6 \times 6 \times 6) \text{ cm}^3 = 216 \text{ cm}^3.$$

$$\text{Surface area} = 6a^2 = (6 \times 6 \times 6) \text{ cm}^2 = 216 \text{ cm}^2.$$

Ex. 8. The surface area of a cube is 1734 sq. cm. Find its volume.

Sol. Let the edge of the cube be a . Then,

$$6a^2 = 1734 \Rightarrow a^2 = 289 \Rightarrow a = 17 \text{ cm}.$$

$$\therefore \text{Volume} = a^3 = (17)^3 \text{ cm}^3 = 4913 \text{ cm}^3.$$

Ex. 9. A rectangular block 6 cm by 12 cm by 15 cm is cut up into an exact number of equal cubes. Find the least possible number of cubes.

$$\text{Sol. Volume of the block} = (6 \times 12 \times 15) \text{ cm}^3 = 1080 \text{ cm}^3.$$

$$\text{Side of the largest cube} = \text{H.C.F. of 6 cm, 12 cm, 15 cm} = 3 \text{ cm}.$$

$$\text{Volume of this cube} = (3 \times 3 \times 3) \text{ cm}^3 = 27 \text{ cm}^3.$$

$$\text{Number of cubes} = \left(\frac{1080}{27} \right) = 40.$$

Ex. 10. A cube of edge 15 cm is immersed completely in a rectangular vessel containing water. If the dimensions of the base of vessel are 20 cm \times 15 cm, find the rise in water level. (R.R.B. 2003)

$$\text{Sol. Increase in volume} = \text{Volume of the cube} = (15 \times 15 \times 15) \text{ cm}^3.$$

$$\therefore \text{Rise in water level} = \left(\frac{\text{Volume}}{\text{Area}} \right) = \left(\frac{15 \times 15 \times 15}{20 \times 15} \right) \text{ cm} = 11.25 \text{ cm}.$$

Ex. 11. Three solid cubes of sides 1 cm, 6 cm and 8 cm are melted to form a new cube. Find the surface area of the cube so formed.

$$\text{Sol. Volume of new cube} = (1^3 + 6^3 + 8^3) \text{ cm}^3 = 729 \text{ cm}^3.$$

$$\text{Edge of new cube} = \sqrt[3]{729} \text{ cm} = 9 \text{ cm}.$$

$$\therefore \text{Surface area of the new cube} = (6 \times 9 \times 9) \text{ cm}^2 = 486 \text{ cm}^2.$$

Ex. 12. If each edge of a cube is increased by 50%, find the percentage increase in its surface area.

Sol. Let original length of each edge = a .

$$\text{Then, original surface area} = 6a^2.$$

$$\text{New edge} = (150\% \text{ of } a) = \left(\frac{150}{100} a \right) = \frac{3a}{2}.$$

$$\text{New surface area} = 6 \times \left(\frac{3a}{2} \right)^2 = \frac{27}{2} a^2.$$

$$\text{Increase percent in surface area} = \left(\frac{15}{2} a^2 \times \frac{1}{6a^2} \times 100 \right) \% = 125\%.$$

Ex. 13. Two cubes have their volumes in the ratio 1 : 27. Find the ratio of their surface areas.

Sol. Let their edges be a and b . Then,

$$\frac{a^3}{b^3} = \frac{1}{27} \text{ or } \left(\frac{a}{b} \right)^3 = \left(\frac{1}{3} \right)^3 \text{ or } \frac{a}{b} = \frac{1}{3}.$$

$$\therefore \text{Ratio of their surface areas} = \frac{6a^2}{6b^2} = \frac{a^2}{b^2} = \left(\frac{a}{b} \right)^2 = \frac{1}{9}, \text{ i.e., } 1:9.$$

Ex. 14. Find the volume, curved surface area and the total surface area of a cylinder with diameter of base 7 cm and height 40 cm.

Sol. Volume = $\pi r^2 h = \left(\frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \times 40 \right) \text{ cm}^3 = 1540 \text{ cm}^3$.

Curved surface area = $2\pi rh = \left(2 \times \frac{22}{7} \times \frac{7}{2} \times 40 \right) \text{ cm}^2 = 880 \text{ cm}^2$.

Total surface area = $2\pi rh + 2\pi r^2 = 2\pi r(h + r)$
 $= \left[2 \times \frac{22}{7} \times \frac{7}{2} \times (40 + 3.5) \right] \text{ cm}^2 = 957 \text{ cm}^2$.

Ex. 15. If the capacity of a cylindrical tank is 1848 m³ and the diameter of its base is 14 m, then find the depth of the tank.

Sol. Let the depth of the tank be h metres. Then,

$$\pi \times (7)^2 \times h = 1848 \Leftrightarrow h = \left(1848 \times \frac{7}{22} \times \frac{1}{7 \times 7} \right) = 12 \text{ m}$$

Ex. 16. 2.2 cubic dm of lead is to be drawn into a cylindrical wire 0.50 cm in diameter. Find the length of the wire in metres.

Sol. Let the length of the wire be h metres. Then,

$$\pi \times \left(\frac{0.50}{2 \times 100} \right)^2 \times h = \frac{2.2}{1000} \Leftrightarrow h = \left(\frac{2.2}{1000} \times \frac{100 \times 100}{0.25 \times 0.25} \times \frac{7}{22} \right) = 112 \text{ m}$$

Ex. 17. How many iron rods, each of length 7 m and diameter 2 cm can be made out of 0.88 cubic metre of iron ? (C.B.I. 1998)

Sol. Volume of 1 rod = $\left(\frac{22}{7} \times \frac{1}{100} \times \frac{1}{100} \times 7 \right) \text{ cu. m} = \frac{11}{5000} \text{ cu. m}$.

Volume of iron = 0.88 cu. m.

Number of rods = $\left(0.88 \times \frac{5000}{11} \right) = 400$.

Ex. 18. The radii of two cylinders are in the ratio 3 : 5 and their heights are in the ratio of 2 : 3. Find the ratio of their curved surface areas.

Sol. Let the radii of the cylinders be $3x, 5x$ and their heights be $2y, 3y$ respectively. Then,

$$\text{Ratio of their curved surface areas} = \frac{2\pi \times 3x \times 2y}{2\pi \times 5x \times 3y} = \frac{2}{5} = 2:5$$

Ex. 19. If 1 cubic cm of cast iron weighs 21 gms, then find the weight of a cast iron pipe of length 1 metre with a bore of 3 cm and in which thickness of the metal is 1 cm.

Sol. Inner radius = $\left(\frac{3}{2} \right) \text{ cm} = 1.5 \text{ cm}$, Outer radius = $(1.5 + 1) = 2.5 \text{ cm}$.

\therefore Volume of iron = $[\pi \times (2.5)^2 \times 100 - \pi \times (1.5)^2 \times 100] \text{ cm}^3$

$$= \frac{22}{7} \times 100 \times [(2.5)^2 - (1.5)^2] \text{ cm}^3 = \left(\frac{8800}{7} \right) \text{ cm}^3$$

\therefore Weight of the pipe = $\left(\frac{8800}{7} \times \frac{21}{1000} \right) \text{ kg} = 26.4 \text{ kg}$.

Ex. 20. Find the slant height, volume, curved surface area and the whole surface area of a cone of radius 21 cm and height 28 cm.

Sol. Here, $r = 21 \text{ cm}$ and $h = 28 \text{ cm}$.

\therefore Slant height, $l = \sqrt{r^2 + h^2} = \sqrt{(21)^2 + (28)^2} = \sqrt{1225} = 35 \text{ cm}$.

$$\text{Volume} = \frac{1}{3} \pi r^2 h = \left(\frac{1}{3} \times \frac{22}{7} \times 21 \times 21 \times 28 \right) \text{ cm}^3 = 12936 \text{ cm}^3.$$

$$\text{Curved surface area} = \pi r l = \left(\frac{22}{7} \times 21 \times 35 \right) \text{ cm}^2 = 2310 \text{ cm}^2.$$

$$\text{Total surface area} = (\pi r l + \pi r^2) = \left(2310 + \frac{22}{7} \times 21 \times 21 \right) \text{ cm}^2 = 3696 \text{ cm}^2.$$

Ex. 21. Find the length of canvas 1.25 m wide required to build a conical tent of base radius 7 metres and height 24 metres.

Sol. Here, $r = 7\text{m}$ and $h = 24\text{ m}$.

$$\text{So, } l = \sqrt{r^2 + h^2} = \sqrt{7^2 + (24)^2} = \sqrt{625} = 25 \text{ m}.$$

$$\text{Area of canvas} = \pi r l = \left(\frac{22}{7} \times 7 \times 25 \right) \text{ m}^2 = 550 \text{ m}^2.$$

$$\therefore \text{Length of canvas} = \left(\frac{\text{Area}}{\text{Width}} \right) = \left(\frac{550}{1.25} \right) \text{ m} = 440 \text{ m}.$$

Ex. 22. The heights of two right circular cones are in the ratio 1 : 2 and the perimeters of their bases are in the ratio 3 : 4. Find the ratio of their volumes.

Sol. Let the radii of their bases be r and R and their heights be h and $2h$ respectively.

$$\text{Then, } \frac{2\pi r}{2\pi R} = \frac{3}{4} \Rightarrow \frac{r}{R} = \frac{3}{4} \Rightarrow R = \frac{4}{3} r.$$

$$\therefore \text{Ratio of volumes} = \frac{\frac{1}{3} \pi r^2 h}{\frac{1}{3} \pi \left(\frac{4}{3} r \right)^2 (2h)} = \frac{9}{32} = 9 : 32.$$

Ex. 23. The radii of the bases of a cylinder and a cone are in the ratio of 3 : 4 and their heights are in the ratio 2 : 3. Find the ratio of their volumes.

Sol. Let the radii of the cylinder and the cone be $3r$ and $4r$ and their heights be $2h$ and $3h$ respectively.

$$\therefore \frac{\text{Volume of cylinder}}{\text{Volume of cone}} = \frac{\pi \times (3r)^2 \times 2h}{\frac{1}{3} \pi \times (4r)^2 \times 3h} = \frac{9}{8} = 9 : 8.$$

Ex. 24. A conical vessel, whose internal radius is 12 cm and height 50 cm, is full of liquid. The contents are emptied into a cylindrical vessel with internal radius 10 cm. Find the height to which the liquid rises in the cylindrical vessel.

Sol. Volume of the liquid in the cylindrical vessel

$$= \text{Volume of the conical vessel}$$

$$= \left(\frac{1}{3} \times \frac{22}{7} \times 12 \times 12 \times 50 \right) \text{ cm}^3 = \left(\frac{22 \times 4 \times 12 \times 50}{7} \right) \text{ cm}^3.$$

Let the height of the liquid in the vessel be h .

$$\text{Then, } \frac{22}{7} \times 10 \times 10 \times h = \frac{22 \times 4 \times 12 \times 50}{7} \text{ or } h = \left(\frac{4 \times 12 \times 50}{10 \times 10} \right) = 24 \text{ cm}.$$

Ex. 25. Find the volume and surface area of a sphere of radius 10.5 cm.

$$\text{Sol. Volume} = \frac{4}{3} \pi r^3 = \left(\frac{4}{3} \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \times \frac{21}{2} \right) \text{ cm}^3 = 4851 \text{ cm}^3.$$

$$\text{Surface area} = 4\pi r^2 = \left(4 \times \frac{22}{7} \times \frac{21}{2} \times \frac{21}{2} \right) \text{ cm}^2 = 1386 \text{ cm}^2.$$

26. RACES AND GAMES

IMPORTANT FACTS

Races: A contest of speed in running, riding, driving, sailing or rowing is called race

Course: The ground or path on which contests are made is called a race course.

Starting Point: The point from which a race begins is known as a starting point.

Winning Point or Goal: The point set to bound a race is called a winning point or a goal.

Winner: The person who first reaches the winning point is called a winner.

Dead Heat Race: If all the persons contesting a race reach the goal exactly at the same time, then the race is said to be a dead heat race.

Start: Suppose A and B are two contestants in a race. If before the start of the race, A is at the starting point and B is ahead of A by 12 metres, then we say that 'A gives B, a start of 12 metres.

To cover a race of 100 metres in this case, A will have to cover 100 metres while B will

have to cover only $(100 - 12) = 88$ metres.

In a 100 m race, 'A can give B 12 m' or 'A can give B a start of 12 m' or 'A beats B by 12 m' means that while A runs 100 m, B runs $(100 - 12) = 88$ m.

Games: 'A game of 100, means that the person among the contestants who scores 100m first is the winner.

If A scores 100 points while B scores only 80 points, then we say that 'A can give B 20 points.

SOLVED EXAMPLES :

Ex. 1. In a km race, A beats B by 28 metres or 7 seconds. Find A's time over the course.

Sol. Clearly, B covers 28 m in 7 seconds.

\therefore B's time over the course = $(\frac{278}{7} \times 1000)$ sec = 250 seconds.

\therefore A's time over the course = $(250 - 7)$ sec = 243 sec = 4 min. 3 sec.

Ex. 2. A runs $1\frac{3}{4}$ times as fast as B. if A gives B a start of 84 m, how far must winning post be so that A and B might reach it at the same time?

Sol. Ratio of the rates of A and B = $\frac{7}{4} : 1 = 7 : 4$.

So, in a race of 7 m, A gains 3m over B.

\therefore 3 m are gained by A in a race of 7 m.

\therefore 84 m are gained by A in a race of $(\frac{7}{3} \times 84)$ m = 196 m.

\therefore Winning post must be 196 m away from the starting point.

Ex. 3. A can run 1 km in 3 min. 10 sec. and B can cover the same distance in 3 min. 20 sec. By what distance can A beat B ?

Soln: Clearly, A beats B by 10 sec.

Distance covered by B in 10 sec. = $(\frac{1000}{200} \times 10)$ m = 50 m.

Therefore A beats B by 50 metres.

Ex .4 . In a 100 m race, A runs at 8km per hour. If A gives B a start of 4 m and still him by 15 seconds, what is the speed of B ?

Sol: Time taken by A to cover 100 m = $(\frac{60 \times 60}{8000}) \times 100$ sec = 45 sec.

B covers $(100 - 4)$ m = 96 m in $(45 + 15)$ sec = 60 sec.

B's speed = $(\frac{96 \times 60 \times 60}{60 \times 1000})$ km/hr = 5.76 km/hr.

Ex. 5. A, B and C are three contestants in a km race. If A can give B a start of 40 m and A can give C a start of 64m how many metre's start can B give C ?

Sol: While A covers 1000 m, B covers $(1000 - 40)$ m = 960 m and

C covers $(1000 - 64)$ m or 936 m.
When B covers 960 m, C covers 936 m.

Ex 6. In a game of 80 points; A can give B 5 points and C 15 points. Then how many points B can give C in a game of 60 ?

Sol. $A : B = 80 : 75$, $A : C = 80 : 65$.

$B/C = (B/A * A/C) = (75 / 80 * 80 / 65) = 15/13 = 60/52 = 60: 52$

Therefore ,In a game of 60, B can give C 8 points.

27. CALENDAR

IMPORTANT FACTS AND FORMULAE

Under this heading we mainly deal with finding the day of the week on a particular given date. The process of finding it lies on obtaining the number of odd days.

I. Odd Days : Number of days more than the complete number of weeks in a given period is the number of odd days during that period.

II. Leap Year : Every year which is divisible by 4 is called a leap year.

Thus, each one of the years 1992, 1996, 2004, 2008, 2012, etc. is a leap year.

Every 4th century is a leap year but no other century is a leap year.

Thus, each one of 400, 800, 1200, 1600, 2000, etc. is a leap year.

None of 1900, 2100, 2200, etc. is a leap year.

An year which is not a leap year is called an ordinary year.

III. (i) An ordinary year has 365 days. (ii) A leap year has 366 days.

IV. Counting of Odd Days :

(i) 1 ordinary year = 365 days = (52 weeks + 1 day).

\therefore An ordinary year has 1 odd day.

(ii) 1 leap year = 366 days = (52 weeks + 2 days).

\therefore A leap year has 2 odd days.

(iii) 100 years = 76 ordinary years + 24 leap years

= [(76 \times 52) weeks + 76 days] + [(24 \times 52) weeks + 48 days]

= 5200 weeks + 124 days = (5217 weeks + 5 days).

\therefore 100 years contain 5 odd days.

200 years contain 10 and therefore 3 odd days.

300 years contain 15 and therefore 1 odd day.

400 years contain (20 + 1) and therefore 0 odd day.

Similarly, each one of 800, 1200, 1600, 2000, etc. contains 0 odd days.

Remark : $(7n + m)$ odd days, where $m < 7$ is equivalent to m odd days.

Thus, 8 odd days = 1 odd day etc.

V.	No. of odd days	0	1	2	3	4	5	6
	Day	Sun.	Mon.	Tues.	Wed.	Thur.	Fri.	Sat.

SOLVED EXAMPLES

Ex. 1. What was the day of the week on 16th July, 1776?

Sol. 16th July, 1776 = (1775 years + Period from 1st Jan., 1776 to 16th July, 1776)

Counting of odd days :

1600 years have 0 odd day. 100 years have 5 odd days.

75 years = (18 leap years + 57 ordinary years)

= [(18 \times 2) + (57 \times 1)] odd days = 93 odd days

= (13 weeks + 2 days) = 2 odd days.

∴ 1775 years have $(0 + 5 + 2)$ odd days = 7 odd days = 0 odd day.

Jan.	Feb.	March	April	May	June	July
31	29	31	30	31	30	16

= 198 days
= (28 weeks + 2 days) = 2 odd days.

∴ Total number of odd days = $(0 + 2)$ = 2. Required day was 'Tuesday'.

Ex. 2. What was the day of the week on 15th August, 1947?

Sol. 15th August, 1947 = (1946 years + Period from 1st Jan., 1947 to 15th Aug., 1947)

Counting of odd days :

1600 years have 0 odd day. 300 years have 1 odd day.

47 years = (11 leap years + 36 ordinary years)

= $[(11 \times 2) + (36 \times 1)]$ odd days = 58 odd days = 2 odd days.

Jan.	Feb.	March	April	May	June	July	Aug.
31	28	31	30	31	30	31	15

= 227 days = (32 weeks + 3 days) = 3 odd days.

Total number of odd days = $(0 + 1 + 2 + 3)$ odd days = 6 odd days.

Hence, the required day was 'Saturday'.

Ex. 3. What was the day of the week on 16th April, 2000?

Sol. 16th April, 2000 = (1999 years + Period from 1st Jan., 2000 to 16th April, 2000)

Counting of odd days :

1600 years have 0 odd day. 300 years have 1 odd day.

99 years = (24 leap years + 75 ordinary years)

= $[(24 \times 2) + (75 \times 1)]$ odd days = 123 odd days

= (17 weeks + 4 days) = 4 odd days.

Jan.	Feb.	March	April
------	------	-------	-------

31 + 29 + 31 + 16 = 107 days = (15 weeks + 2 days) = 2 odd days.

Total number of odd days = $(0 + 1 + 4 + 2)$ odd days = 7 odd days = 0 odd day.

Hence, the required day was 'Sunday'.

Ex. 4. On what dates of July 2004 did Monday fall?

Sol. Let us find the day on 1st July, 2004.

2000 years have 0 odd day. 3 ordinary years have 3 odd days.

Jan.	Feb.	March	April	May	June	July
31	29	31	30	31	30	1

= 183 days = (26 weeks + 1 day) = 1 odd day.

Total number of odd days = $(0 + 3 + 1)$ odd days = 4 odd days.

∴ 1st July 2004 was 'Thursday'.

Thus, 1st Monday in July 2004 was on 5th July.

Hence, during July 2004, Monday fell on 5th, 12th, 19th and 26th.

Ex. 5. Prove that the calendar for the year 2003 will serve for the year 2014.

Sol. In order that the calendar for the year 2003 and 2014 be the same, 1st January of both the years must be on the same day of the week.

For this, the number of odd days between 31st Dec., 2002 and 31st Dec., 2013 must be the same.

We know that an ordinary year has 1 odd day and a leap year has 2 odd days.

During this period, there are 3 leap years, namely 2004, 2008 and 2012 and 8 ordinary years.

Total number of odd days = $(6 + 8)$ days = 0 odd day.

Hence, the calendar for 2003 will serve for the year 2014.

Ex. 6. Prove that any date in March of a year is the same day of the week as the corresponding date in November that year.

Sol. We will show that the number of odd days between last day of February and last day of October is zero.

March	April	May	June	July	Aug.	Sept.	Oct.
31	30	31	30	31	31	30	31
							= 241 days = 35 weeks = 0 odd day.

∴ Number of odd days during this period = 0.

Thus, 1st March of an year will be the same day as 1st November of that year.

Hence, the result follows.

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28. CLOCKS

IMPORTANT FACTS

The face or dial of a watch is a circle whose circumference is divided into 60 equal parts, called minute spaces.

A clock has two hands, the smaller one is called the *hour hand* or *short hand* while the larger one is called the *minute hand* or *long hand*.

- (i) In 60 minutes, the minute hand gains 55 minutes on the hour hand.
- (ii) In every hour, both the hands coincide once.
- (iii) The hands are in the same straight line when they are coincident or opposite to each other.
- (iv) When the two hands are at right angles, they are 15 minute spaces apart.
- (v) When the hands are in opposite directions, they are 30 minute spaces apart.
- (vi) Angle traced by hour hand in 12 hrs = 360° .
- (vii) Angle traced by minute hand in 60 min. = 360° .

Too Fast and Too Slow : If a watch or a clock indicates 8.15, when the correct time is 8, it is said to be 15 minutes too fast.

On the other hand, if it indicates 7.45, when the correct time is 8, it is said to be 15 minutes too slow.

SOLVED EXAMPLES

Ex. 1. Find the angle between the hour hand and the minute hand of a clock when the time is 3.25.

Sol. Angle traced by the hour hand in 12 hours = 360° .

Angle traced by it in 3 hrs 25 min. i.e. $\frac{41}{12}$ hrs = $\left(\frac{360}{12} \times \frac{41}{12}\right)^\circ = 102\frac{1}{2}^\circ$.

Angle traced by minute hand in 60 min. = 360° .

Angle traced by it in 25 min. = $\left(\frac{360}{60} \times 25\right)^\circ = 150^\circ$.

\therefore Required angle = $\left(150^\circ - 102\frac{1}{2}^\circ\right) = 47\frac{1}{2}^\circ$.

Ex. 2. At what time between 2 and 3 o'clock will the hands of a clock be together?

Sol. At 2 o'clock, the hour hand is at 2 and the minute hand is at 12, i.e. they are 10 min. spaces apart.

To be together, the minute hand must gain 10 minutes over the hour hand.

Now, 55 minutes are gained by it in 60 min.

\therefore 10 minutes will be gained in $\left(\frac{60}{55} \times 10\right)$ min. = $10\frac{10}{11}$ min.

\therefore The hands will coincide at $10\frac{10}{11}$ min. past 2.

Ex. 3. At what time between 4 and 5 o'clock will the hands of a clock be at right angle ?

Sol. At 4 o'clock, the minute hand will be 20 min. spaces behind the hour hand.
Now, when the two hands are at right angles, they are 15 min. spaces apart.
So, they are at right angles in following two cases.

Case I. When minute hand is 15 min. spaces behind the hour hand :

In this case min. hand will have to gain $(20 - 15) = 5$ minute spaces.

55 min. spaces are gained by it in 60 min.

5 min. spaces will be gained by it in $\left(\frac{60}{55} \times 5\right)$ min. $= 5\frac{5}{11}$ min.

\therefore They are at right angles at $5\frac{5}{11}$ min. past 4.

Case II. When the minute hand is 15 min. spaces ahead of the hour hand :

To be in this position, the minute hand will have to gain $(20 + 15) = 35$ minute spaces.

55 min. spaces are gained in 60 min.

35 min. spaces are gained in $\left(\frac{60}{55} \times 35\right)$ min. $= 38\frac{2}{11}$ min.

\therefore They are at right angles at $38\frac{2}{11}$ min. past 4.

Ex. 4. Find at what time between 8 and 9 o'clock will the hands of a clock be in the same straight line but not together.

Sol. At 8 o'clock, the hour hand is at 8 and the minute hand is at 12, i.e. the two hands are 20 min. spaces apart.

To be in the same straight line but not together they will be 30 minute spaces apart.

So, the minute hand will have to gain $(30 - 20) = 10$ minute spaces over the hour hand.

55 minute spaces are gained in 60 min.

10 minute spaces will be gained in $\left(\frac{60}{55} \times 10\right)$ min. $= 10\frac{10}{11}$ min.

\therefore The hands will be in the same straight line but not together at $10\frac{10}{11}$ min. past 8.

Ex. 5. At what time between 5 and 6 o'clock are the hands of a clock 3 minutes apart ?

Sol. At 5 o'clock, the minute hand is 25 min. spaces behind the hour hand.

Case I. Minute hand is 3 min. spaces behind the hour hand.

In this case, the minute hand has to gain $(25 - 3) = 22$ minute spaces.

55 min. are gained in 60 min.

22 min. are gained in $\left(\frac{60}{55} \times 22\right)$ min. $= 24$ min.

\therefore The hands will be 3 min. apart at 24 min. past 5.

Case II. Minute hand is 3 min. spaces ahead of the hour hand.

In this case, the minute hand has to gain $(25 + 3) = 28$ minute spaces.

55 min. are gained in 60 min.

28 min. are gained in $\left(\frac{60}{55} \times 28\right) = 31\frac{5}{11}$ min.

\therefore The hands will be 3 min. apart at $31\frac{5}{11}$ min. past 5.

Ex. 6. The minute hand of a clock overtakes the hour hand at intervals of 65 minutes of the correct time. How much a day does the clock gain or lose?

Sol. In a correct clock, the minute hand gains 55 min. spaces over the hour hand in 60 minutes.

To be together again, the minute hand must gain 60 minutes over the hour hand.
55 min. are gained in 60 min.

$$60 \text{ min. are gained in } \left(\frac{60}{55} \times 60 \right) \text{ min.} = 65 \frac{5}{11} \text{ min.}$$

But, they are together after 65 min.

$$\therefore \text{Gain in 65 min.} = \left(65 \frac{5}{11} - 65 \right) = \frac{5}{11} \text{ min.}$$

$$\text{Gain in 24 hours} = \left(\frac{5}{11} \times \frac{60 \times 24}{65} \right) \text{ min.} = 10 \frac{10}{43} \text{ min.}$$

$$\therefore \text{The clock gains } 10 \frac{10}{43} \text{ minutes in 24 hours.}$$

Ex. 7. A watch which gains uniformly, is 5 min. slow at 8 o'clock in the morning on Sunday and it is 5 min. 48 sec. fast at 8 p.m. on following Sunday. When was it correct?

Sol. Time from 8 a.m. on Sunday to 8 p.m. on following Sunday = 7 days 12 hours
= 180 hours

$$\therefore \text{The watch gains } \left(5 + 5 \frac{4}{5} \right) \text{ min. or } \frac{54}{5} \text{ min. in 180 hrs.}$$

$$\text{Now } \frac{54}{5} \text{ min. are gained in 180 hrs.}$$

$$\therefore 5 \text{ min. are gained in } \left(180 \times \frac{5}{54} \times 5 \right) \text{ hrs.} = 83 \text{ hrs } 20 \text{ min.} = 3 \text{ days } 11 \text{ hrs } 20 \text{ min.}$$

\therefore Watch is correct 3 days 11 hrs 20 min. after 8 a.m. of Sunday.

\therefore It will be correct at 20 min. past 7 p.m. on Wednesday.

Ex. 8. A clock is set right at 5 a.m. The clock loses 16 minutes in 24 hours. What will be the true time when the clock indicates 10 p.m. on 4th day?

Sol. Time from 5 a.m. on a day to 10 p.m. on 4th day = 89 hours.

Now 23 hrs 44 min. of this clock = 24 hours of correct clock.

$$\therefore \frac{356}{15} \text{ hrs of this clock} = 24 \text{ hours of correct clock.}$$

$$89 \text{ hrs of this clock} = \left(24 \times \frac{15}{356} \times 89 \right) \text{ hrs of correct clock.}$$

$$= 90 \text{ hrs of correct clock.}$$

So, the correct time is 11 p.m.

29. STOCK AND SHARES

To start a big business or an industry, a large amount of money is needed. It is beyond the capacity of one or two persons to arrange such a huge amount. However, some persons associate together to form a company. They, then, draft a proposal, issue a prospectus (in the name of the company), explaining the plan of the project and invite the public to invest money in this project. They, thus, pool up the funds from the public, by assigning them **shares** of the company.

IMPORTANT FACTS AND FORMULAE

1. **Stock-capital** : The total amount of money needed to run the company is called the **stock-capital**.

2. **Shares or Stock** : The whole capital is divided into small units, called **shares** or **stock**.

For each investment, the company issues a **share-certificate**, showing the value of each share and the number of shares held by a person.

The person who subscribes in shares or stock is called a **share holder** or **stock holder**.

3. **Dividend** : The annual profit distributed among share holders is called **dividend**. Dividend is paid annually as per share or as a percentage.

4. **Face Value** : The value of a share or stock printed on the share-certificate is called its **Face Value** or **Nominal Value** or **Par Value**.

5. **Market Value** : The stocks of different companies are sold and bought in the open market through brokers at stock-exchanges. A share (or stock) is said to be :

(i) **At premium** or **Above par**, if its market value is more than its face value.

(ii) **At par**, if its market value is the same as its face value.

(iii) **At discount** or **Below par**, if its market value is less than its face value.

Thus, if a Rs. 100 stock is quoted at a premium of 16, then market value of the stock = Rs. $(100 + 16)$ = Rs. 116.

Likewise, if a Rs. 100 stock is quoted at a discount of 7, then market value of the stock = Rs. $(100 - 7)$ = Rs. 93.

6. **Brokerage** : The broker's charge is called **brokerage**.

(i) When stock is purchased, brokerage is added to the cost price.

(ii) When stock is sold, brokerage is subtracted from the selling price.

Remember :

(i) The face value of a share always remains the same.

(ii) The market value of a share changes from time to time.

(iii) Dividend is always paid on the face value of a share.

(iv) Number of shares held by a person

$$= \frac{\text{Total Investment}}{\text{Investment in 1 share}} = \frac{\text{Total Income}}{\text{Income from 1 share}} = \frac{\text{Total Face Value}}{\text{Face value of 1 share}}$$

Thus, by a Rs. 100, 9% stock at 120, we mean that :

- (i) Face Value (N.V.) of stock = Rs. 100.
- (ii) Market Value (M.V.) of stock = Rs. 120.
- (iii) Annual dividend on 1 share = 9% of face value = 9% of Rs. 100 = Rs. 9.
- (iv) An investment of Rs. 120 gives an annual income of Rs. 9.
- (v) Rate of interest p.a. = Annual income from an investment of Rs. 100
$$= \left(\frac{9}{120} \times 100 \right) \% = 7\frac{1}{2}\%.$$

SOLVED EXAMPLES

Ex. 1. Find the cost of :

- (i) Rs. 7200, 8% stock at 90;
- (ii) Rs. 4500, 8.5% stock at 4 premium;
- (iii) Rs. 6400, 10% stock at 15 discount.

Sol. (i) Cost of Rs. 100 stock = Rs. 90.

$$\text{Cost of Rs. 7200 stock} = \text{Rs. } \left(\frac{90}{100} \times 7200 \right) = \text{Rs. 6480.}$$

(ii) Cost of Rs. 100 stock = Rs. (100 + 4) = Rs. 104.

$$\text{Cost of Rs. 4500 stock} = \text{Rs. } \left(\frac{104}{100} \times 4500 \right) = \text{Rs. 4680.}$$

(iii) Cost of Rs. 100 stock = Rs. (100 - 15) = Rs. 85.

$$\text{Cost of Rs. 6400 stock} = \text{Rs. } \left(\frac{85}{100} \times 6400 \right) = \text{Rs. 5440.}$$

Ex. 2. Find the cash required to purchase Rs. 3200, $7\frac{1}{2}\%$ stock at 107 (brokerage $\frac{1}{2}\%$).

Sol. Cash required to purchase Rs. 100 stock = Rs. $\left(107 + \frac{1}{2} \right) = \text{Rs. } \frac{215}{2}.$

$$\text{Cash required to purchase Rs. 3200 stock} = \text{Rs. } \left(\frac{215}{2} \times \frac{1}{100} \times 3200 \right) = \text{Rs. 3440.}$$

Ex. 3. Find the cash realised by selling Rs. 2440, 9.5% stock at 4 discount (brokerage $\frac{1}{4}\%$).

Sol. By selling Rs. 100 stock, cash realised = Rs. $\left[(100 - 4) - \frac{1}{4} \right] = \text{Rs. } \frac{383}{4}.$

$$\text{By selling Rs. 2400 stock, cash realised} = \text{Rs. } \left(\frac{383}{4} \times \frac{1}{100} \times 2400 \right) = \text{Rs. 2298.}$$

Ex. 4. Find the annual income derived from Rs. 2500, 8% stock at 106.

Sol. Income from Rs. 100 stock = Rs. 8.

$$\text{Income from Rs. 2500 stock} = \text{Rs. } \left(\frac{8}{100} \times 2500 \right) = \text{Rs. 200.}$$

Ex. 5. Find the annual income derived by investing Rs. 6800 in 10% stock at 136.

Sol. By investing Rs. 136, income obtained = Rs. 10.

By investing Rs. 6800, income obtained = Rs. $\left(\frac{10}{136} \times 6800\right)$ = Rs. 500.

Ex. 6. Which is better investment? $7\frac{1}{2}\%$ stock at 105 or $6\frac{1}{2}\%$ stock at 94.

Sol. Let the investment in each case be Rs. (105×94) .

Case I : $7\frac{1}{2}\%$ stock at 105 :

On investing Rs. 105, income = Rs. $\frac{15}{2}$.

On investing Rs. (105×94) , income = Rs. $\left(\frac{15}{2} \times \frac{1}{105} \times 105 \times 94\right)$ = Rs. 705.

Case II : $6\frac{1}{2}\%$ stock at 94 :

On investing Rs. 94, income = Rs. $\frac{13}{2}$.

On investing Rs. (105×94) , income = Rs. $\left(\frac{13}{2} \times \frac{1}{94} \times 105 \times 94\right)$ = Rs. 682.50.

Clearly, the income from $7\frac{1}{2}\%$ stock at 105 is more.

Hence, the investment in $7\frac{1}{2}\%$ stock at 105 is better.

Ex. 7. Find the cost of 96 shares of Rs. 10 each at $\frac{3}{4}$ discount, brokerage being $\frac{1}{4}$ per share. (L.I.C. 2003)

Sol. Cost of 1 share = Rs. $\left[\left(10 - \frac{3}{4}\right) + \frac{1}{4}\right]$ = Rs. $\frac{19}{2}$.

Cost of 96 shares = Rs. $\left(\frac{19}{2} \times 96\right)$ = Rs. 912.

Ex. 8. Find the income derived from 88 shares of Rs. 25 each at 5 premium, brokerage being $\frac{1}{4}$ per share and the rate of dividend being $7\frac{1}{2}\%$ per annum. Also, find the rate of interest on the investment.

Sol. Cost of 1 share = Rs. $\left(25 + 5 + \frac{1}{4}\right)$ = Rs. $\frac{121}{4}$.

Cost of 88 shares = Rs. $\left(\frac{121}{4} \times 88\right)$ = Rs. 2662.

\therefore Investment made = Rs. 2662.

Face value of 88 shares = Rs. (88×25) = Rs. 2200.

Dividend on Rs. 100 = $\frac{15}{2}$.

Dividend on Rs. 2200 = Rs. $\left(\frac{15}{2} \times \frac{1}{100} \times 2200\right)$ = Rs. 165.

\therefore Income derived = Rs. 165.

Rate of interest on investment = $\left(\frac{165}{2662} \times 100\right)$ = 6.2%.

Ex. 9. A man buys Rs. 25 shares in a company which pays 9% dividend. The money invested is such that it gives 10% on investment. At what price did he buy the shares?

Sol. Suppose he buys each share for Rs. x .

$$\text{Then, } \left(25 \times \frac{9}{100}\right) = \left(x \times \frac{10}{100}\right) \text{ or } x = 22.50.$$

\therefore Cost of each share = Rs. 22.50.

Ex. 10. A man sells Rs. 5000, 12% stock at 156 and invests the proceeds partly in 8% stock at 90 and 9% stock at 108. He thereby increases his income by Rs. 70. How much of the proceeds were invested in each stock?

$$\text{Sol. S.P. of Rs. 5000 stock} = \text{Rs. } \left(\frac{156}{100} \times 5000\right) = \text{Rs. 7800.}$$

$$\text{Income from this stock} = \text{Rs. } \left(\frac{12}{100} \times 5000\right) = \text{Rs. 600.}$$

Let investment in 8% stock be x and that in 9% stock = $(7800 - x)$.

$$\therefore \left(x \times \frac{8}{90}\right) + (7800 - x) \times \frac{9}{108} = (600 + 70)$$

$$\Leftrightarrow \frac{4x}{45} + \frac{7800 - x}{12} = 670 \quad \Leftrightarrow 16x + 117000 - 15x = (670 \times 180) \quad \Leftrightarrow x = 3600.$$

\therefore Money invested in 8% stock at 90 = Rs. 3600.

Money invested in 9% at 108 = Rs. $(7800 - 3600) = \text{Rs. 4200.}$

30. PERMUTATIONS AND COMBINATIONS

IMPORTANT FACTS AND FORMULAE

Factorial Notation : Let n be a positive integer. Then, factorial n , denoted by $n!$ or $n!$ is defined as :

$$n! = n(n-1)(n-2) \dots 3.2.1.$$

Examples : (i) $5! = (5 \times 4 \times 3 \times 2 \times 1) = 120$; (ii) $4! = (4 \times 3 \times 2 \times 1) = 24$ etc.

We define, $0! = 1$.

Permutations : The different arrangements of a given number of things by taking some or all at a time, are called permutations.

Ex. 1. All permutations (or arrangements) made with the letters a, b, c by taking two at a time are (ab, ba, ac, ca, bc, cb) .

Ex. 2. All permutations made with the letters a, b, c , taking all at a time are : $(abc, acb, bac, bca, cab, cba)$.

Number of Permutations : Number of all permutations of n things, taken r at a time, is given by :

$${}^n P_r = n(n-1)(n-2) \dots (n-r+1) = \frac{n!}{(n-r)!}$$

Examples : (i) ${}^6 P_2 = (6 \times 5) = 30$. (ii) ${}^7 P_3 = (7 \times 6 \times 5) = 210$.

Cor. Number of all permutations of n things, taken all at a time = $n!$

An Important Result : If there are n objects of which p_1 are alike of one kind; p_2 are alike of another kind; p_3 are alike of third kind and so on and p_r are alike of r th kind, such that $(p_1 + p_2 + \dots + p_r) = n$.

Then, **number of permutations of these n objects is :**

$$\frac{n!}{(p_1!) \cdot (p_2!) \dots (p_r!)}$$

Combinations : Each of the different groups or selections which can be formed by taking some or all of a number of objects, is called a combination.

Ex. 1. Suppose we want to select two out of three boys A, B, C . Then, possible selections are AB, BC and CA .

Note that AB and BA represent the same selection.

Ex. 2. All the combinations formed by a, b, c , taking two at a time are ab, bc, ca .

Ex. 3. The only combination that can be formed of three letters a, b, c taken all at a time is abc .

Ex. 4. Various groups of 2 out of four persons A, B, C, D are :

$$AB, AC, AD, BC, BD, CD.$$

Ex. 5. Note that ab and ba are two different permutations but they represent the same combination.

Number of Combinations : The number of all combinations of n things, taken r at a time is :

$${}^nC_r = \frac{n!}{(r!)(n-r)!} = \frac{n(n-1)(n-2)\dots \text{to } r \text{ factors}}{r!}$$

Note that : ${}^nC_n = 1$ and ${}^nC_0 = 1$.

An Important Result : ${}^nC_r = {}^nC_{(n-r)}$.

Example : (i) ${}^{11}C_4 = \frac{(11 \times 10 \times 9 \times 8)}{(4 \times 3 \times 2 \times 1)} = 330$.

(ii) ${}^{16}C_{13} = {}^{16}C_{(16-13)} = {}^{16}C_3 = \frac{16 \times 15 \times 14}{3!} = \frac{16 \times 15 \times 14}{3 \times 2 \times 1} = 560$.

SOLVED EXAMPLES

Ex. 1. Evaluate : $\frac{30!}{28!}$

Sol. We have, $\frac{30!}{28!} = \frac{30 \times 29 \times (28!)}{28!} = (30 \times 29) = 870$.

Ex. 2. Find the value of (i) ${}^{60}P_3$ (ii) 4P_4

Sol. (i) ${}^{60}P_3 = \frac{60!}{(60-3)!} = \frac{60!}{57!} = \frac{60 \times 59 \times 58 \times (57!)}{57!} = (60 \times 59 \times 58) = 205320$.

(ii) ${}^4P_4 = 4! = (4 \times 3 \times 2 \times 1) = 24$.

Ex. 3. Find the value of (i) ${}^{10}C_3$ (ii) ${}^{100}C_{98}$ (iii) ${}^{50}C_{50}$

Sol. (i) ${}^{10}C_3 = \frac{10 \times 9 \times 8}{3!} = \frac{10 \times 9 \times 8}{3 \times 2 \times 1} = 120$.

(ii) ${}^{100}C_{98} = {}^{100}C_{(100-98)} = {}^{100}C_2 = \left(\frac{100 \times 99}{2 \times 1} \right) = 4950$.

(iii) ${}^{50}C_{50} = 1$ [$\because {}^nC_n = 1$]

Ex. 4. How many words can be formed by using all letters of the word 'BIHAR' ?

Sol. The word BIHAR contains 5 different letters.

\therefore Required number of words = ${}^5P_5 = 5! = (5 \times 4 \times 3 \times 2 \times 1) = 120$.

Ex. 5. How many words can be formed by using all the letters of the word 'DAUGHTER' so that the vowels always come together ?

Sol. Given word contains 8 different letters. When the vowels AUE are always together, we may suppose them to form an entity, treated as one letter.

Then, the letters to be arranged are DGHTR (AUE).

These 6 letters can be arranged in ${}^6P_6 = 6! = 720$ ways.

The vowels in the group (AUE) may be arranged in $3! = 6$ ways.

\therefore Required number of words = $(720 \times 6) = 4320$.

Ex. 6. How many words can be formed from the letters of the word 'EXTRA', so that the vowels are never together ?

Sol. The given word contains 5 different letters.

Taking the vowels EA together, we treat them as one letter.

Then, the letters to be arranged are XTR (EA).

These letters can be arranged in $4! = 24$ ways.

The vowels EA may be arranged amongst themselves in $2! = 2$ ways.

Number of words, each having vowels together $= (24 \times 2) = 48$.

Total number of words formed by using all the letters of the given words

$$= 5! = (5 \times 4 \times 3 \times 2 \times 1) = 120.$$

Number of words, each having vowels never together $= (120 - 48) = 72$.

Ex. 7. How many words can be formed from the letters of the word 'DIRECTOR' so that the vowels are always together?

Sol. In the given word, we treat the vowels IEO as one letter.

Thus, we have DRCTR (IEO).

This group has 6 letters of which R occurs 2 times and others are different.

$$\text{Number of ways of arranging these letters} = \frac{6!}{2!} = 360.$$

Now 3 vowels can be arranged among themselves in $3! = 6$ ways.

\therefore Required number of ways $= (360 \times 6) = 2160$.

Ex. 8. In how many ways can a cricket eleven be chosen out of a batch of 15 players?

Sol. Required number of ways $= {}^{15}C_{11} = {}^{15}C_{(15-11)} = {}^{15}C_4$

$$= \frac{15 \times 14 \times 13 \times 12}{4 \times 3 \times 2 \times 1} = 1365.$$

Ex. 9. In how many ways, a committee of 5 members can be selected from 6 men and 5 ladies, consisting of 3 men and 2 ladies?

Sol. (3 men out of 6) and (2 ladies out of 5) are to be chosen.

$$\therefore \text{Required number of ways} = ({}^6C_3 \times {}^5C_2) = \left(\frac{6 \times 5 \times 4}{3 \times 2 \times 1} \times \frac{5 \times 4}{2 \times 1} \right) = 200.$$

31. PROBABILITY

IMPORTANT FACTS AND FORMULAE

1. **Experiment** : An operation which can produce some well-defined outcomes is called an experiment.

2. **Random Experiment** : An experiment in which all possible outcomes are known and the exact output cannot be predicted in advance, is called a random experiment.

Examples of Performing a Random Experiment :

(i) Rolling an unbiased dice.

(ii) Tossing a fair coin.

(iii) Drawing a card from a pack of well-shuffled cards.

(iv) Picking up a ball of certain colour from a bag containing balls of different colours.

Details :

(i) When we throw a coin. Then either a Head (H) or a Tail (T) appears.

(ii) A dice is a solid cube, having 6 faces, marked 1, 2, 3, 4, 5, 6 respectively. When we throw a die, the outcome is the number that appears on its upper face.

(iii) A pack of cards has 52 cards.

It has 13 cards of each suit, namely **Spades, Clubs, Hearts and Diamonds**.

Cards of spades and clubs are **black cards**.

Cards of hearts and diamonds are **red cards**.

There are 4 honours of each suit.

These are **Aces, Kings, Queens and Jacks**.

These are called **face cards**.

3. **Sample Space** : When we perform an experiment, then the set S of all possible outcomes is called the **Sample Space**.

Examples of Sample Spaces :

(i) In tossing a coin, $S = \{H, T\}$.

(ii) If two coins are tossed, then $S = \{HH, HT, TH, TT\}$.

(iii) In rolling a dice, we have, $S = \{1, 2, 3, 4, 5, 6\}$.

4. **Event** : Any subset of a sample space is called an event.

5. **Probability of Occurrence of an Event** :

Let S be the sample space and let E be an event.

Then, $E \subset S$.

$$\therefore P(E) = \frac{n(E)}{n(S)}$$

6. **Results on Probability :**

(i) $P(S) = 1$ (ii) $0 \leq P(E) \leq 1$ (iii) $P(\phi) = 0$

(iv) For any events A and B, we have :

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

(v) If \bar{A} denotes (not-A), then $P(\bar{A}) = 1 - P(A)$.

SOLVED EXAMPLES

Ex. 1. In a throw of a coin, find the probability of getting a head.

Sol. Here $S = \{H, T\}$ and $E = \{H\}$.

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{1}{2}$$

Ex. 2. Two unbiased coins are tossed. What is the probability of getting at most one head?

Sol. Here $S = \{HH, HT, TH, TT\}$.

Let $E =$ event of getting at most one head.

$\therefore E = \{TT, HT, TH\}$.

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{3}{4}$$

Ex. 3. An unbiased die is tossed. Find the probability of getting a multiple of 3.

Sol. Here $S = \{1, 2, 3, 4, 5, 6\}$.

Let E be the event of getting a multiple of 3.

Then, $E = \{3, 6\}$.

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

Ex. 4. In a simultaneous throw of a pair of dice, find the probability of getting a total more than 7.

Sol. Here, $n(S) = (6 \times 6) = 36$.

Let $E =$ Event of getting a total more than 7

$= \{(2, 6), (3, 5), (3, 6), (4, 4), (4, 5), (4, 6), (5, 3), (5, 4), (5, 5), (5, 6), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)\}$.

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{15}{36} = \frac{5}{12}$$

Ex. 5. A bag contains 6 white and 4 black balls. Two balls are drawn at random. Find the probability that they are of the same colour.

Sol. Let S be the sample space. Then,

$$n(S) = \text{Number of ways of drawing 2 balls out of } (6+4) = {}^{10}C_2 = \frac{(10 \times 9)}{(2 \times 1)} = 45.$$

Let $E =$ Event of getting both balls of the same colour. Then,

$n(E) =$ Number of ways of drawing (2 balls out of 6) or (2 balls out of 4)

$$= ({}^6C_2 + {}^4C_2) = \frac{(6 \times 5)}{(2 \times 1)} + \frac{(4 \times 3)}{(2 \times 1)} = (15 + 6) = 21.$$

$$\therefore P(E) = \frac{n(E)}{n(S)} = \frac{21}{45} = \frac{7}{15}$$

Ex. 6. Two dice are thrown together. What is the probability that the sum of the numbers on the two faces is divisible by 4 or 6?

Sol. Clearly, $n(S) = 6 \times 6 = 36$.

Let E be the event that the sum of the numbers on the two faces is divisible by 4 or 6. Then

$E = \{(1, 3), (1, 5), (2, 2), (2, 4), (2, 6), (3, 1), (3, 3), (3, 5), (4, 2), (4, 4), (5, 1), (5, 3), (6, 2), (6, 6)\}$

$\therefore n(E) = 14$.

$$\text{Hence, } P(E) = \frac{n(E)}{n(S)} = \frac{14}{36} = \frac{7}{18}$$

Ex. 7. Two cards are drawn at random from a pack of 52 cards. What is the probability that either both are black or both are queens?

Sol. We have $n(S) = {}^{52}C_2 = \frac{(52 \times 51)}{(2 \times 1)} = 1326$.

Let A = event of getting both black cards;

B = event of getting both queens.

$\therefore A \cap B$ = event of getting queens of black cards.

$\therefore n(A) = {}^{26}C_2 = \frac{(26 \times 25)}{(2 \times 1)} = 325$, $n(B) = {}^4C_2 = \frac{(4 \times 3)}{(2 \times 1)} = 6$ and $n(A \cap B) = {}^2C_2 = 1$.

$\therefore P(A) = \frac{n(A)}{n(S)} = \frac{325}{1326}$; $P(B) = \frac{n(B)}{n(S)} = \frac{6}{1326}$ and $P(A \cap B) = \frac{n(A \cap B)}{n(S)} = \frac{1}{1326}$.

$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B) = \left(\frac{325}{1326} + \frac{6}{1326} - \frac{1}{1326} \right) = \frac{330}{1326} = \frac{55}{221}$.

32. TRUE DISCOUNT

IMPORTANT CONCEPTS

Suppose a man has to pay Rs. 156 after 4 years and the rate of interest is 14% per annum. Clearly, Rs. 100 at 14% will amount to Rs. 156 in 4 years. So, the payment of Rs. 100 now will clear off the debt of Rs. 156 due 4 years hence. We say that :

Sum due = Rs. 156 due 4 years hence;

Present Worth (P.W.) = Rs. 100;

True Discount (T.D.) = Rs. (156 - 100) = Rs. 56 = (Sum due) - (P.W.).

We define : **T.D. = Interest on P.W.**

Amount = (P.W.) + (T.D.).

Interest is reckoned on P.W. and true discount is reckoned on the amount.

IMPORTANT FORMULAE

Let rate = $R\%$ per annum and Time = T years. Then,

$$1. \text{ P.W.} = \frac{100 \times \text{Amount}}{100 + (R \times T)} = \frac{100 \times \text{T.D.}}{R \times T} \quad 2. \text{ T.D.} = \frac{(\text{P.W.}) \times R \times T}{100} = \frac{\text{Amount} \times R \times T}{100 + (R \times T)}$$

$$3. \text{ Sum} = \frac{(\text{S.I.}) \times (\text{T.D.})}{(\text{S.I.}) - (\text{T.D.})} \quad 4. (\text{S.I.}) - (\text{T.D.}) = \text{S.I. on T.D.}$$

$$5. \text{ When the sum is put at compound interest, then } \text{P.W.} = \frac{\text{Amount}}{\left(1 + \frac{R}{100}\right)^T}$$

SOLVED EXAMPLES

Ex. 1. Find the present worth of Rs. 930 due 3 years hence at 8% per annum. Also find the discount.

$$\text{Sol. P.W.} = \frac{100 \times \text{Amount}}{100 + (R \times T)} = \text{Rs.} \left[\frac{100 \times 930}{100 + (8 \times 3)} \right] = \text{Rs.} \left(\frac{100 \times 930}{124} \right) = \text{Rs.} 750.$$

$$\text{T.D.} = (\text{Amount}) - (\text{P.W.}) = \text{Rs.} (930 - 750) = \text{Rs.} 180.$$

Ex. 2. The true discount on a bill due 9 months hence at 12% per annum is Rs. 540. Find the amount of the bill and its present worth.

Sol. Let amount be Rs. x . Then,

$$\frac{x \times R \times T}{100 + (R \times T)} = \text{T.D.} \Rightarrow \frac{x \times 12 \times \frac{3}{4}}{100 + \left(12 \times \frac{3}{4}\right)} = 540 \Rightarrow x = \left(\frac{540 \times 109}{9} \right) = \text{Rs.} 6540.$$

$$\therefore \text{Amount} = \text{Rs.} 6540.$$

$$\text{P.W.} = \text{Rs.} (6540 - 540) = \text{Rs.} 6000.$$

Ex. 3. The true discount on a certain sum of money due 3 years hence is Rs. 250 and the simple interest on the same sum for the same time and at the same rate is Rs. 375. Find the sum and the rate percent.

Sol. T.D. = Rs. 250 and S.I. = Rs. 375.

$$\therefore \text{Sum due} = \frac{\text{S.I.} \times \text{T.D.}}{(\text{S.I.}) - (\text{T.D.})} = \text{Rs.} \left(\frac{375 \times 250}{375 - 250} \right) = \text{Rs.} 750.$$

$$\text{Rate} = \left(\frac{100 \times 375}{750 \times 3} \right) \% = 16\frac{2}{3}\%.$$

Ex. 4. The difference between the simple interest and true discount on a certain sum of money for 6 months at $12\frac{1}{2}\%$ per annum is Rs. 25. Find the sum.

Sol. Let the sum be Rs. x . Then,

$$\text{T.D.} = \frac{x \times \frac{25}{2} \times \frac{1}{2}}{100 + \left(\frac{25}{2} \times \frac{1}{2}\right)} = \left(x \times \frac{25}{4} \times \frac{4}{425} \right) = \frac{x}{17}.$$

$$\text{S.I.} = \left(x \times \frac{25}{2} \times \frac{1}{2} \times \frac{1}{100} \right) = \frac{x}{16}.$$

$$\therefore \frac{x}{16} - \frac{x}{17} = 25 \Rightarrow 17x - 16x = 25 \times 16 \times 17 \Rightarrow x = 6800.$$

Hence, sum due = Rs. 6800.

Ex. 5. A bill falls due in 1 year. The creditor agrees to accept immediate payment of the half and to defer the payment of the other half for 2 years. By this arrangement he gains Rs. 40. What is the amount of the bill, if the money be worth $12\frac{1}{2}\%$?

Sol. Let the sum be Rs. x . Then,

$$\left[\frac{x}{2} + \frac{\frac{x}{2} \times 100}{100 + \left(\frac{25}{2} \times 2\right)} \right] - \frac{x \times 100}{100 + \left(\frac{25}{2} \times 1\right)} = 40 \Rightarrow \frac{x}{2} + \frac{2x}{5} - \frac{8x}{9} = 40 \Rightarrow x = 3600.$$

\therefore Amount of the bill = Rs. 3600.

33. BANKER'S DISCOUNT

IMPORTANT CONCEPTS

Banker's Discount : Suppose a merchant A buys goods worth, say Rs. 10,000 from another merchant B at a credit of say 5 months. Then, B prepares a bill, called the bill of exchange. A signs this bill and allows B to withdraw the amount from his bank account after exactly 5 months.

The date exactly after 5 months is called *nominally due date*. Three days (known as *grace days*) are added to it to get a date, known as *legally due date*.

Suppose B wants to have the money before the legally due date. Then he can have the money from the banker or a broker, who deducts S.I. on the face value (i.e., Rs. 10,000 in this case) for the period from the date on which the bill was discounted (i.e., paid by the banker) and the legally due date. This amount is known as **Banker's Discount (B.D.)**.

Thus, **B.D. is the S.I. on the face value for the period from the date on which the bill was discounted and the legally due date.**

Banker's Gain (B.G.) = (B.D.) - (T.D.) for the unexpired time.

Note : When the date of the bill is not given, grace days are not to be added.

IMPORTANT FORMULAE

$$1. \text{B.D.} = \text{S.I. on bill for unexpired time.}$$

$$2. \text{B.G.} = (\text{B.D.}) - (\text{T.D.}) = \text{S.I. on T.D.} = \frac{(\text{T.D.})^2}{\text{P.W.}}$$

$$3. \text{T.D.} = \sqrt{\text{P.W.} \times \text{B.G.}}$$

$$4. \text{B.D.} = \left(\frac{\text{Amount} \times \text{Rate} \times \text{Time}}{100} \right)$$

$$5. \text{T.D.} = \left[\frac{\text{Amount} \times \text{Rate} \times \text{Time}}{100 + (\text{Rate} \times \text{Time})} \right]$$

$$6. \text{Amount} = \left(\frac{\text{B.D.} \times \text{T.D.}}{\text{B.D.} - \text{T.D.}} \right)$$

$$7. \text{T.D.} = \left(\frac{\text{B.G.} \times 100}{\text{Rate} \times \text{Time}} \right)$$

SOLVED EXAMPLES

Ex. 1. A bill for Rs. 6000 is drawn on July 14 at 5 months. It is discounted on 5th October at 10%. Find the banker's discount, true discount, banker's gain and the money that the holder of the bill receives.

Sol. Face value of the bill = Rs. 6000.

Date on which the bill was drawn = July 14 at 5 months.

Nominally due date = December 14. Legally due date = December 17.

Date on which the bill was discounted = October 5.

Unexpired time : Oct.

Nov.

Dec.

$$26 + 30 + 17 = 73 \text{ days} = \frac{1}{5} \text{ year.}$$

$$\therefore \text{B.D.} = \text{S.I. on Rs. 6000 for } \frac{1}{5} \text{ year} = \text{Rs. } \left(6000 \times 10 \times \frac{1}{5} \times \frac{1}{100} \right) = \text{Rs. 120.}$$

$$\text{T.D.} = \text{Rs. } \left[\frac{6000 \times 10 \times \frac{1}{5}}{100 + \left(10 \times \frac{1}{5} \right)} \right] = \text{Rs. } \left(\frac{12000}{102} \right) = \text{Rs. 117.64.}$$

$$\therefore \text{B.G.} = (\text{B.D.}) - (\text{T.D.}) = \text{Rs. } (120 - 117.64) = \text{Rs. 2.36.}$$

Money received by the holder of the bill = Rs. $(6000 - 120) = \text{Rs. 5880.}$

Ex. 2. If the true discount on a certain sum due 6 months hence at 15% is Rs. 120, what is the banker's discount on the same sum for the same time and at the same rate?

$$\text{Sol. B.G.} = \text{S.I. on T.D.} = \text{Rs. } \left(120 \times 15 \times \frac{1}{2} \times \frac{1}{100} \right) = \text{Rs. 9.}$$

$$\therefore (\text{B.D.}) - (\text{T.D.}) = \text{Rs. 9.}$$

$$\therefore \text{B.D.} = \text{Rs. } (120 + 9) = \text{Rs. 129.}$$

Ex. 3. The banker's discount on Rs. 1800 at 12% per annum is equal to the true discount on Rs. 1872 for the same time at the same rate. Find the time.

$$\text{Sol. S.I. on Rs. 1800} = \text{T.D. on Rs. 1872.}$$

$$\therefore \text{P.W. of Rs. 1872 is Rs. 1800.}$$

$$\therefore \text{Rs. 72 is S.I. on Rs. 1800 at 12\%.}$$

$$\therefore \text{Time} = \left(\frac{100 \times 72}{12 \times 1800} \right) \text{ year} = \frac{1}{3} \text{ year} = 4 \text{ months.}$$

Ex. 4. The banker's discount and the true discount on a sum of money due 8 months hence are Rs. 120 and Rs. 110 respectively. Find the sum and the rate percent.

$$\text{Sol. Sum} = \left(\frac{\text{B.D.} \times \text{T.D.}}{\text{B.D.} - \text{T.D.}} \right) = \text{Rs. } \left(\frac{120 \times 110}{120 - 110} \right) = \text{Rs. 1320.}$$

Since B.D. is S.I. on sum due, so S.I. on Rs. 1320 for 8 months is Rs. 120.

$$\therefore \text{Rate} = \left(\frac{100 \times 120}{1320 \times \frac{2}{3}} \right) \% = 13\frac{7}{11} \%.$$

Ex. 5. The present worth of a bill due sometime hence is Rs. 1100 and the true discount on the bill is Rs. 110. Find the banker's discount and the banker's gain.

$$\text{Sol. T.D.} = \sqrt{\text{P.W.} \times \text{B.G.}}$$

$$\therefore \text{B.G.} = \frac{(\text{T.D.})^2}{\text{P.W.}} = \text{Rs. } \left(\frac{110 \times 110}{1100} \right) = \text{Rs. 11.}$$

$$\therefore \text{B.D.} = (\text{T.D.} + \text{B.G.}) = \text{Rs. } (110 + 11) = \text{Rs. 121.}$$

Ex. 6. The banker's discount on Rs. 1650 due a certain time hence is Rs. 165. Find the true discount and the banker's gain.

$$\text{Sol. Sum} = \frac{\text{B.D.} \times \text{T.D.}}{\text{B.D.} - \text{T.D.}} = \frac{\text{B.D.} \times \text{T.D.}}{\text{B.G.}}$$

$$\therefore \frac{\text{T.D.}}{\text{B.G.}} = \frac{\text{Sum}}{\text{B.D.}} = \frac{1650}{165} = \frac{10}{1}$$

Thus, if B.G. is Re 1, T.D. = Rs. 10.

$$\text{If B.D. is Rs. 11, T.D.} = \text{Rs. 10. If B.D. is Rs. 165, T.D.} = \text{Rs. } \left(\frac{10}{11} \times 165 \right) = \text{Rs. 150.}$$

$$\text{And, B.G.} = \text{Rs. } (165 - 150) = \text{Rs. 15.}$$

Ex. 7. What rate percent does a man get for his money when in discounting a bill due 10 months hence, he deducts 10% of the amount of the bill?

Sol. Let, amount of the bill = Rs. 100. Money deducted = Rs. 10.

Money received by the holder of the bill = Rs. $(100 - 10) =$ Rs. 90.

\therefore S.I. on Rs. 90 for 10 months = Rs. 10.

$$\therefore \text{Rate} = \left(\frac{100 \times 10}{90 \times \frac{10}{12}} \right) \% = 13\frac{1}{3}\%.$$

34. HEIGHTS AND DISTANCES

IMPORTANT FACTS AND FORMULAE

1. We already know that :

In a rt. angled $\triangle OAB$, where $\angle BOA = \theta$,

$$(i) \sin \theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{AB}{OB};$$

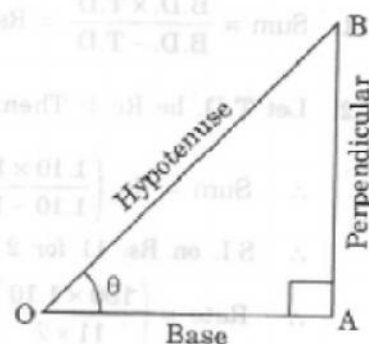
$$(ii) \cos \theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{OA}{OB};$$

$$(iii) \tan \theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{AB}{OA};$$

$$(iv) \operatorname{cosec} \theta = \frac{1}{\sin \theta} = \frac{OB}{AB};$$

$$(v) \sec \theta = \frac{1}{\cos \theta} = \frac{OB}{OA};$$

$$(vi) \cot \theta = \frac{1}{\tan \theta} = \frac{OA}{AB};$$



2. Trigonometrical Identities :

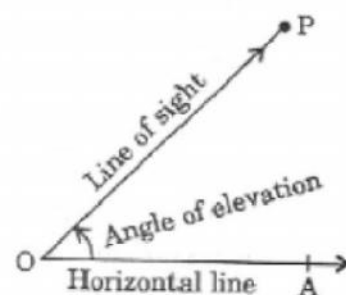
$$(i) \sin^2 \theta + \cos^2 \theta = 1. \quad (ii) 1 + \tan^2 \theta = \sec^2 \theta. \quad (iii) 1 + \cot^2 \theta = \operatorname{cosec}^2 \theta.$$

3. Values of T-ratios :

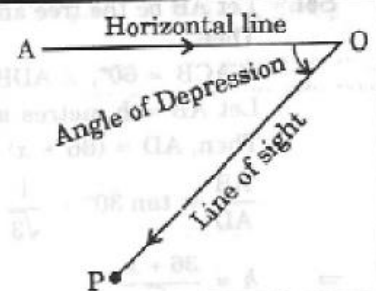
θ	0°	$(\pi/6)$ 30°	$(\pi/4)$ 45°	$(\pi/3)$ 60°	$(\pi/2)$ 90°
$\sin \theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos \theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan \theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	not defined

4. **Angle of Elevation** : Suppose a man from a point O looks up at an object P, placed above the level of his eye. Then, the angle which the line of sight makes with the horizontal through O, is called the angle of elevation of P as seen from O.

\therefore Angle of elevation of P from O = $\angle AOP$.



5. **Angle of Depression** : Suppose a man from a point O looks down at an object P, placed below the level of his eye, then the angle which the line of sight makes with the horizontal through O, is called the angle of depression of P as seen from O.



SOLVED EXAMPLES

Ex. 1. If the height of a pole is $2\sqrt{3}$ metres and the length of its shadow is 2 metres, find the angle of elevation of the sun.

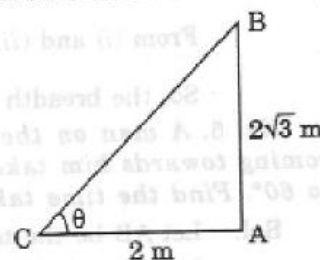
Sol. Let AB be the pole and AC be its shadow.

Let angle of elevation, $\angle ACB = \theta$.

Then, AB = $2\sqrt{3}$ m, AC = 2 m.

$$\tan \theta = \frac{AB}{AC} = \frac{2\sqrt{3}}{2} = \sqrt{3} \Rightarrow \theta = 60^\circ.$$

So, the angle of elevation is 60° .



Ex. 2. A ladder leaning against a wall makes an angle of 60° with the ground. If the length of the ladder is 19 m, find the distance of the foot of the ladder from the wall.

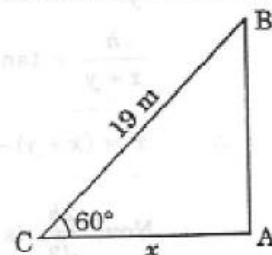
Sol. Let AB be the wall and BC be the ladder.

Then, $\angle ACB = 60^\circ$ and BC = 19 m.

Let AC = x metres

$$\frac{AC}{BC} = \cos 60^\circ \Rightarrow \frac{x}{19} = \frac{1}{2} \Rightarrow x = \frac{19}{2} = 9.5.$$

\therefore Distance of the foot of the ladder from the wall = 9.5 m.



Ex. 3. The angle of elevation of the top of a tower at a point on the ground is 30° . On walking 24 m towards the tower, the angle of elevation becomes 60° . Find the height of the tower.

Sol. Let AB be the tower and C and D be the points of observation. Then,

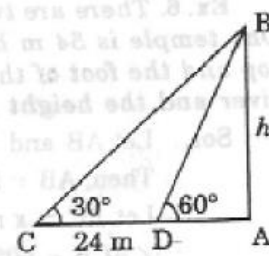
$$\frac{AB}{AD} = \tan 60^\circ = \sqrt{3} \Rightarrow AD = \frac{AB}{\sqrt{3}} = \frac{h}{\sqrt{3}}.$$

$$\frac{AB}{AC} = \tan 30^\circ = \frac{1}{\sqrt{3}} \Rightarrow AC = AB \times \sqrt{3} = h\sqrt{3}.$$

$$CD = (AC - AD) = \left(h\sqrt{3} - \frac{h}{\sqrt{3}} \right).$$

$$\therefore h\sqrt{3} - \frac{h}{\sqrt{3}} = 24 \Rightarrow h = 12\sqrt{3} = (12 \times 1.73) = 20.76.$$

Hence, the height of the tower is 20.76 m.



Ex. 4. A man standing on the bank of a river observes that the angle subtended by a tree on the opposite bank is 60° . When he retires 36 m from the bank, he finds the angle to be 30° . Find the breadth of the river.

Sol. Let AB be the tree and AC be the river. Let C and D be the two positions of the man.

Then,

$\angle ACB = 60^\circ$, $\angle ADB = 30^\circ$ and $CD = 36$ m.

Let $AB = h$ metres and $AC = x$ metres.

Then, $AD = (36 + x)$ metres.

$$\frac{AB}{AD} = \tan 30^\circ = \frac{1}{\sqrt{3}} \Rightarrow \frac{h}{36 + x} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow h = \frac{36 + x}{\sqrt{3}} \quad \dots(i)$$

$$\frac{AB}{AC} = \tan 60^\circ = \sqrt{3} \Rightarrow \frac{h}{x} = \sqrt{3}$$

$$\Rightarrow h = \sqrt{3} x \quad \dots(ii)$$

$$\text{From (i) and (ii), we get : } \frac{36 + x}{\sqrt{3}} = \sqrt{3} x \Rightarrow x = 18 \text{ m.}$$

So, the breadth of the river = 18 m.

Ex. 5. A man on the top of a tower, standing on the seashore finds that a boat coming towards him takes 10 minutes for the angle of depression to change from 30° to 60° . Find the time taken by the boat to reach the shore from this position.

Sol. Let AB be the tower and C and D be the two positions of the boat.

Let $AB = h$, $CD = x$ and $AD = y$.

$$\frac{h}{y} = \tan 60^\circ = \sqrt{3} \Rightarrow y = \frac{h}{\sqrt{3}}$$

$$\frac{h}{x + y} = \tan 30^\circ = \frac{1}{\sqrt{3}} \Rightarrow x + y = \sqrt{3} h.$$

$$\therefore x = (x + y) - y = \left(\sqrt{3} h - \frac{h}{\sqrt{3}} \right) = \frac{2h}{\sqrt{3}}$$

Now, $\frac{2h}{\sqrt{3}}$ is covered in 10 min.

$$\therefore \frac{h}{\sqrt{3}} \text{ will be covered in } \left(10 \times \frac{\sqrt{3}}{2h} \times \frac{h}{\sqrt{3}} \right) = 5 \text{ min.}$$

Hence, required time = 5 minutes.

Ex. 6. There are two temples, one on each bank of a river, just opposite to each other. One temple is 54 m high. From the top of this temple, the angles of depression of the top and the foot of the other temple are 30° and 60° respectively. Find the width of the river and the height of the other temple.

Sol. Let AB and CD be the two temples and AC be the river.

Then, $AB = 54$ m.

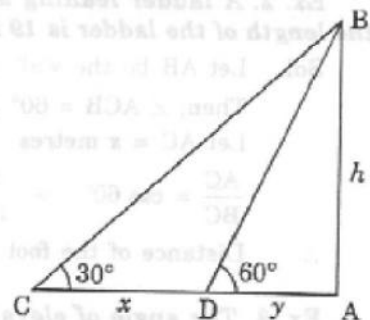
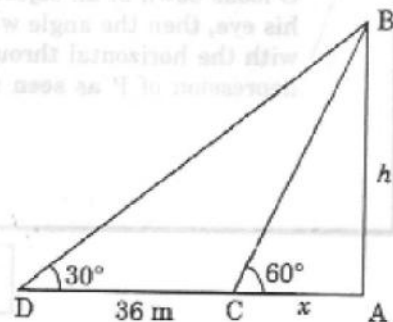
Let $AC = x$ metres and $CD = h$ metres.

$\angle ACB = 60^\circ$, $\angle EDB = 30^\circ$.

$$\frac{AB}{AC} = \tan 60^\circ = \sqrt{3}$$

$$\Rightarrow AC = \frac{AB}{\sqrt{3}} = \frac{54}{\sqrt{3}} = \left(\frac{54}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \right) = 18\sqrt{3} \text{ m.}$$

$$DE = AC = 18\sqrt{3} \text{ m.}$$



10. A man on the top of a vertical observation tower observes a car moving at a uniform speed coming directly towards it. It takes 12 minutes for the angle of depression to change from 30° to 45° , how soon after this will the car reach the observation tower?

$$\Rightarrow \frac{BE}{DE} = \tan 30^\circ = \frac{1}{\sqrt{3}} \quad \Rightarrow \quad BE = \left(18\sqrt{3} \times \frac{1}{\sqrt{3}} \right) = 18 \text{ m.}$$

$$\therefore CD = AE = AB - BE = (54 - 18) \text{ m} = 36 \text{ m.}$$

$$\text{So, Width of the river} = AC = 18\sqrt{3} \text{ m} = (18 \times 1.73) \text{ m} = 31.14 \text{ m.}$$

$$\text{Height of the other temple} = CD = 18 \text{ m.}$$

36. TABULATION

This section comprises of questions in which certain data regarding common disciplines as production over a period of a few years: imports, exports, incomes of employees in a factory, students applying for and qualifying in certain field of study etc. are given in the form of a table. The candidate is required to understand the given information and thereafter answer the given questions on the basis of comparative analysis of the **data**.

Thus, here the data collected by the investigator are arranged in a systematic form in a table called the *tabular form*. In order to avoid some heads again and again, tables are made consisting of horizontal lines called *rows* and vertical lines called *columns* with distinctive heads, known as *captions*. Units of measurements are given with the captions.

SOLVED EXAMPLES

The following table gives the sales of batteries manufactured by a company over the years. Study the table and answer the questions that follow:

(S.B.I.P.O. 1998)

NUMBER OF DIFFERENT TYPES OF BATTERIES SOLD BY A COMPANY OVER THE YEARS (NUMBERS IN THOUSANDS)

TYPES OF BATTERIES

Year	4AH	7AH	32AH	35AH	55AH	TOTAL
1992	75	144	114	102	108	543
1993	90	126	102	84	426	528
1994	96	114	75	105	135	525
1995	105	90	150	90	75	510
1996	90	75	135	75	90	465
1997	105	60	165	45	120	495
1998	115	85	160	100	145	605

1. The total sales of all the seven years is the maximum for which battery ?
 (a) 4AH (b) 7AH (c) 32AH (d) 35AH (e) 55AH
2. What is the difference in the number of 35AH batteries sold in 1993 and 1997 ?
 (a) 24000 (b) 28000 (c) 35000 (d) 39000 (e) 42000
3. The percentage of 4AH batteries sold to the total number of batteries sold was maximum in the year:
 (a) 1994 (b) 1995 (c) 1996 (d) 1997 (e) 1998
4. In the case of which battery there was a continuous decrease in sales from 1992 to 1997 ?
 (a) 4AH (b) 7AH (c) 32AH (d) 35AH (e) 55AH
5. What was the approximate percentage increase in the sales of 55AH batteries in 1998 compared to that in 1992 ?
 (a) 28% (b) 31% (c) 33% (d) 34% (e) 37%

Sol. 1. (c) : The total sales (in thousands) of all the seven years for various batteries are:

For 4AH = $75 + 90 + 96 + 105 + 90 + 105 + 115 = 676$

For 7AH = $144 + 126 + 114 + 90 + 75 + 60 + 85 = 694$

For 32AH = $114 + 102 + 75 + 150 + 135 + 165 + 160 = 901$

For 35AH = $102 + 84 + 105 + 90 + 75 + 45 + 100 = 601$

For 55AH = $108 + 126 + 135 + 75 + 90 + 120 + 145 = 799$.

Clearly, sales are maximum in case of 32AH batteries.

2. (d) : Required difference = $[(84 - 45) \times 1000] = 39000$.

3. (d) : The percentages of sales of 4AH batteries to the total sales in different years are:

For 1992 = $(75 \times 100 / 543)\% = 13.81\%$

For 1993 = $(90 \times 100 / 528)\% = 17.05\%$

For 1994 = $(96 \times 100 / 465)\% = 19.35\%$

For 1995 = $(105 \times 100 / 495)\% = 20.59\%$

For 1996 = $(96 \times 100 / 465)\% = 19.35\%$

For 1997 = $(105 \times 100 / 495)\% = 21.21\%$

For 1998 = $(115 \times 100 / 605)\% = 19.01\%$

Clearly, the percentage is maximum in 1997.

4. (b) : From the table it is clear that the sales of 7AH batteries have

been decreasing continuously from 1992 to 1997.

5. (d) : Required Percentage $= (145-108)/108 \times 100 \% = 34.26\% = 34\%$.

Ex 2: Study the following table carefully and answer these questions:

NUMBER OF CANDIDATES APPEARED AND QUALIFIED IN A COMPETITIVE EXAMINATION FROM DIFFERENT STATES OVER THE YEAR

	1997		1998		1999		2000		2001	
	App.	Qual.	App.	Qual.	App.	Qual.	App.	Qual.	App.	Qual.
M	5200	720	8500	980	7400	850	6800	775	9500	1125
N	7500	840	9200	1050	8450	920	9200	980	8800	1020
P	6400	780	8800	1020	7800	890	8750	1010	9750	1250
Q	8100	950	9500	1240	8700	980	9700	1200	8950	995
R	7800	870	7600	940	9800	1350	7600	945	7990	885

1. Combining the states P and Q, together in 1998, what is the percentage of the candidates qualified to that of the candidates appeared?

(8) 10.87% (b) 11.49% (c) 12.35% (d) 12.54% (e) 13.50%

2. The percentage of the total number of qualified candidates to the total number appeared candidates among all the five states in 1999 is :

(a) 11.49% (b) 11.84% (c) 12.21% (d) 12.57% (e) 12.7a1

3. What is the percentage of candidates qualified from State N for all the years together, over the candidates appeared from State N during all the years together?

(a) 12.36% (b) 12.16% (c) 11.47% (d) 11.15% (e) None of these

4. What is the average of candidates who appeared from State Q during the given years?

(8) 8700 (b) 8760 (c) 8810 (d) 8920 (e) 8990

5 . In which of the given years the number of candidates appeared from State P has maximum percentage of qualified candidates?

(8) 1997 (b) 1998 (c) 1999 (d) 2000 (e) 2001

6. Total number of candidates qualified from all the states together in 1997

is approximately what percentage of the total number of candidates qualified from all the states together in 1998 ?

- (8) 72% (b) 77% (c) 80% (d) 83% (e) 86%

$$\text{Sol.1.(c) Required Percentage} = \frac{(1020+1240)}{(8800+9500)} * 100\% = \frac{(2260*100)}{18300}\%$$

$$= 12.35\%$$

$$\text{Required Percentage} = \frac{(850+920+890+980+1350)}{(7400+8450+7800+8700+9800)} * 100\%$$

$$= \frac{(4990*100)}{42150}\%$$

$$= 11.84\%$$

$$(e) : \text{Required Percentage} = \frac{(840+1050+920+980+1020)}{(7500+9200+8450+9200+8800)} * 100\%$$

$$= \frac{(4810*100)}{43150}\%$$

$$= 11.15\%$$

$$4. (e) \text{ Required average} = \frac{(8100+9500+8700+9700+8950)}{5}$$

$$= 44950/5$$

$$= 8990$$

5. (e) : The percentages of candidates qualified to candidates appeared from State P during different years are:

$$\text{For 1997} = \frac{780}{6400} * 100\% = 12.19\%$$

$$\text{for 1998} = \frac{1020*100}{8800}\% = 11.59\%$$

$$\text{For 1999} = \frac{890*100}{7800}\% = 11.41\%;$$

$$\text{For 2000} = \frac{1010*100}{8750}\% = 11.54\%.$$

$$\text{For 2001} = \frac{1250*100}{9750}\% = 12.82\%$$

∴ Maximum percentage is for the year 2001.

$$6. (c) : \text{Required Percentage} = \frac{(720 + 840 + 780 + 950 + 870)}{980+1050+1020+1240+940} \times 100$$

$$= 80\%$$

Ex. 3. The following table gives the percentage of marks obtained by seven students in six , different subjects in an examination. Study the table and answer the questions based on it. The numbers in the brackets give the maximum marks in each subject.
(Bank P.O. 2003)

(Max. marks)	Maths	Chemistry	Physics	Geography	History	Computer Science
Student	(160)	(130)	(120)	(100)	(60)	(40)
Ayush	90	50	90	60	70	80
Aman	100	80	80	40	80	70
Sajal	90	60	70	70	90	70
Rohit	80	65	80	80	60	60
Muskan	80	65	85	95	50	90
Tanvi	70	75	65	85	40	60
Tharun	65	35	50	77	80	80

- What was the aggregate of marks obtained by Sajal in all the six subjects?
(a) 409 (b) 419 (c) 429 (d) 439 (e) 449
 - What is the overall percentage of Thrun?
(a) 52.5% (b) 55% (c) 60% (d) 63% (e) 64.5%
 - What are the average marks obtained by all the seven students in Physics? (rounded off to two digits after decimal)
(a) 77.26 (b) 89.14 (c) 91.37 (d) 96.11 (e) 103.21
 - The number of students who obtained 60% and above marks in all the subjects is :
(a) 1 (b) 2 (c) 3 (d) None (e) None of these
 - In which subject is the overall percentage the best?
(a) History (b) Maths (c) Physics (d) Chemistry (e) Geography
- Sol. 1.. (e) : Aggregate marks obtained by Sajal

$$= [(90\% \text{ of } 150) + (60\% \text{ of } 130) + (70\% \text{ of } 120) + (70\% \text{ of } 100) + (90\% \text{ of } 60) + (70\% \text{ of } 40)] = 135 + 78 + 84 + 70 + 54 + 28 = 449.$$

2. (c) : Aggregate marks obtained by Tarun

$$= [(65\% \text{ of } 150) + (35\% \text{ of } 130) + (50\% \text{ of } 120) + (77\% \text{ of } 100) + (80\% \text{ of } 60) + (80\% \text{ of } 40)] = 97.5 + 45.5 + 60 + 77 + 48 + 32 = 360.$$

Total maximum marks (of all the six subjects)

$$= (150 + 130 + 120 + 100 + 60 + 40) = 600.$$

$$\text{Overall percentage of Tarun} = \frac{360 \times 100}{600} \% = 60\%.$$

3. (b) : Average marks obtained in Physics by all the seven students

$$= \frac{1}{7} [(90\% \text{ of } 120) + (80\% \text{ of } 120) + (70\% \text{ of } 120) + (80\% \text{ of } 120) + (85\% \text{ of } 120) + (65\% \text{ of } 120) + (50\% \text{ of } 120)]$$

$$= \frac{1}{7} [(90 + 80 + 70 + 80 + 85 + 65 + 50)\% \text{ of } 120]$$

$$= \frac{1}{7} [520\% \text{ of } 120] = 89.14.$$

4. (b) : From the table it is clear that Sajal and Rohit have 60% or more marks in each of the six subjects.

6. (b) : We shall find the overall percentage (for all the seven students) with respect to each subject.

The overall percentage for any subject is equal to the average of percentages obtained by all the seven students since the maximum marks for any subject is the same for all the students.

Therefore, overall percentage for:

$$(i) \text{ Maths} = \frac{1}{7} (90 + 100 + 90 + 80 + 80 + 70 + 65)\%$$

$$= \frac{1}{7} (575)\% = 82.14\%.$$

$$(ii) \text{ Chemistry} = \frac{1}{7} (50 + 80 + 60 + 65 + 65 + 75 + 35)\%$$

$$= \frac{1}{7} (430)\% = 61.43\%.$$

$$\begin{aligned} \text{(iii) Physics} &= \left[\frac{1}{7} (90 + 80 + 70 + 80 + 85 + 65 + 50) \right] \% \\ &= \left[\frac{1}{7} (520) \right] \% = 74.29\%. \end{aligned}$$

$$\begin{aligned} \text{(iv) Geography} &= \left[\frac{1}{7} (60 + 40 + 70 + 80 + 95 + 85 + 77) \right] \% \\ &= \left[\frac{1}{7} (507) \right] \% = 72.43\%. \end{aligned}$$

$$\begin{aligned} \text{(v) History} &= \left[\frac{1}{7} (70 + 80 + 90 + 60 + 50 + 40 + 80) \right] \% \\ &= \left[\frac{1}{7} (470) \right] \% = 67.14\%. \end{aligned}$$

$$\begin{aligned} \text{(vi) Computer Science} &= \left[\frac{1}{7} (80 + 70 + 70 + 60 + 90 + 60 + 80) \right] \% \\ &= \left[\frac{1}{7} (510) \right] \% = 72.86\%. \end{aligned}$$

Clearly; this. percentage is highest for Maths.

ex.4. Study the following table carefully and answer the questions given below: (Bank P.O. 2001)

CLASSIFICATION OF 100 STUDENTS BASED ON THE MARKS OBTAINED BY THEM IN PHYSICS AND CHEMISTRY IN AN EXAMINATION

Marks out Of 50 Subject	40 and above	30 and Above	20 and above	10 and above	0 and above
physics	9	32	80	92	100
chemistry	4	21	66	81	100
(aggregate Average)	7	27	73	87	100

- The number of students scoring less than 40% marks in aggregate is :
 (a) 13 (b) 19 (c) 20 (d) 27 (e) 34
- If at least 60% marks in Physics are required for pursuing higher studies in Physics, how many students will be eligible to pursue higher studies in Physics?
 (a) 27 (b) 32 (c) 34 (d) 41 (e) 68
- What is the difference between the number of students passed with 30 as

cut-off marks in Chemistry and those passed with :JUas cut-off marks in aggregate?

- (a) 3 (b) 4 (c) 5 (d) 6 (e) 7

4. The percentage of the number of students getting at least 60% marks in Chemistry over those getting at least 40% marks in aggregate, is approximately:

- (a) 21% (b) 27% (c) 29% (d) 31% (e) 34%

5. If it is known that at least 23 students were eligible for a Symposium on Chemistry the minimum qualifying marks in Chemistry for eligibility to Symposium would lie in the range:

- (a) 40-50 (b) 30-40 (c) 20-30 (d) Below 20

Sol. 1. (d) : We have 40% of 50 = $\frac{40}{100} \times 50 = 20$.

\therefore Required number = Number of students scoring less than 20 marks in aggregate
 $= 100 - \text{number of students scoring 20 and above marks in aggregate} = 100 - 73 = 27$.

2. (b) : We have 60% of 50 = $\frac{60}{100} \times 50 = 30$.

\therefore Required number = Number of students scoring 30 and above mark in Physics = 32.

3. (d) : Required difference = (Number of students scoring 30 and above in mark in Chemistry) (Number of students scoring 30 and above marks in aggregate) = $27 - 21 = 6$.

4. (c) : Number of students getting at least 60% marks in Chemistry
 $= \text{Number of students getting 30 and above marks in Chemistry} = 21$.
 Number of students getting at least 40% marks in aggregate
 $= \text{Number of students getting 20 and above marks in aggregate} = 73$.
 \therefore Required Percentage = $\frac{21}{73} \times 100\% = 28.77\% \approx 29\%$.

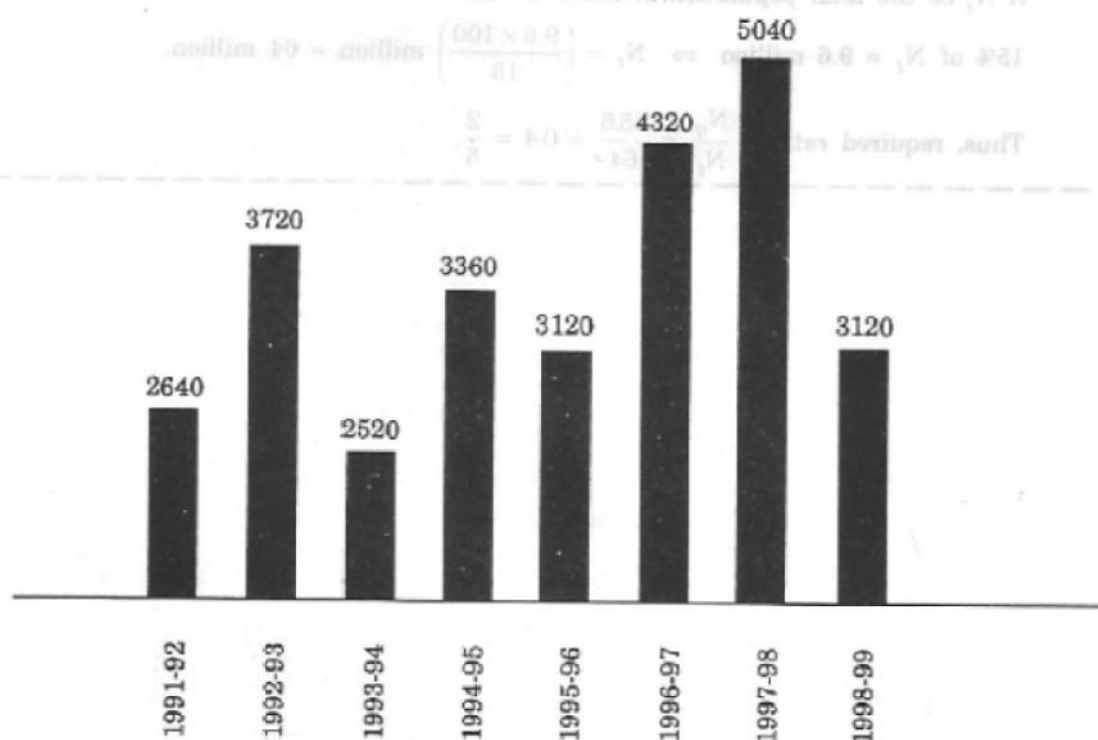
6. (c) : Since 66 students get 20 and above marks in Chemistry and out of these 21 students get 30 and above marks, therefore to select top 35 students in Chemistry, the qualifying marks should lie in the range 20-30.

37. BAR GRAPHS

This section comprises of questions in which the data collected in a particular discipline are represented in the form of vertical or horizontal bars drawn by selecting a particular scale. One of the parameters is plotted on the horizontal axis and the other on the vertical axis. The candidate is required to understand the given information and thereafter answer the given questions on the basis of data analysis.

Ex. 1. The bar graph given below shows the foreign exchange reserves of a country (in million US \$) from 1991-92 to 1998-99. Answer the questions based on this graph. (Bank P.O. 2001)

FOREIGN EXCHANGE RESERVES OF A COUNTRY
(in million US \$)



- The foreign exchange reserves in 1997-98 was how many times that in 1994-95 ?
(a) 0.7 (b) 1.2 (c) 1.4 (d) 1.5 (e) 1.8
- What was the percentage increase in the foreign exchange reserves in 1997-98 over 1993-94 ?
(a) 100 (b) 150 (c) 200 (d) 620 (e) 2520
- For which year, the percent increase of foreign exchange reserves over the previous year, is the highest ?
(a) 1992-93 (b) 1993-94 (c) 1994-95 (d) 1996-97 (e) 1997-98

4. The foreign exchange reserves in 1996-97 were approximately what percent of the average foreign exchange reserves over the period under review ?

- (a) 95% (b) 110% (c) 115% (d) 125% (e) 140%

5. The ratio of the number of years, in which the foreign exchange reserves are above the average reserves, to those in which the reserves are below the average reserves, is :

- (a) 2 : 6 (b) 3 : 4 (c) 3 : 5 (d) 4 : 4 (e) 5 : 3

Sol. 1. (d) : Required ratio = $\frac{5040}{3360} = 1.5$.

2. (a) : Foreign exchange reserves in 1997-98 = 5040 million US \$

Foreign exchange reserves in 1993-94 = 2520 million US \$.

\therefore Increase = $(5040 - 2520) = 2520$ million US \$.

\therefore Percentage increase = $\left(\frac{2520}{2520} \times 100 \right) \% = 100\%$.

3. (a) : There is an increase in foreign exchange reserves during the years 1992-93, 1994-95, 1996-97 and 1997-98 as compared to previous year (as shown by bar-graph).

The percentage increase in reserves during these years compared to previous year are :

$$(i) \text{ For } 1992-93 = \left[\frac{(3720 - 2640)}{2640} \times 100 \right] \% = 40.91\%$$

$$(ii) \text{ For } 1994-95 = \left[\frac{(3360 - 2520)}{2520} \times 100 \right] \% = 33.33\%$$

$$(iii) \text{ For } 1996-97 = \left[\frac{(4320 - 3120)}{3120} \times 100 \right] \% = 38.46\%$$

$$(iv) \text{ For } 1997-98 = \left[\frac{(5040 - 4320)}{4320} \times 100 \right] \% = 16.67\%$$

Clearly, the percentage increase over previous year is highest for 1992-93.

4. (d) : Average foreign exchange reserves over the given period

$$= \left[\frac{1}{8} \times (2640 + 3720 + 2520 + 3360 + 3120 + 4320 + 5040 + 3120) \right] \text{ million US \$}$$

= 3480 million US \$.

Foreign exchange reserves in 1996-97 = 4320 million US \$.

$$\therefore \text{ Required Percentage} = \left(\frac{4320}{3480} \times 100 \right) \% = 124.14\% \approx 125\%.$$

5. (c) : Average foreign exchange reserves over the given period = 3480 million US \$.

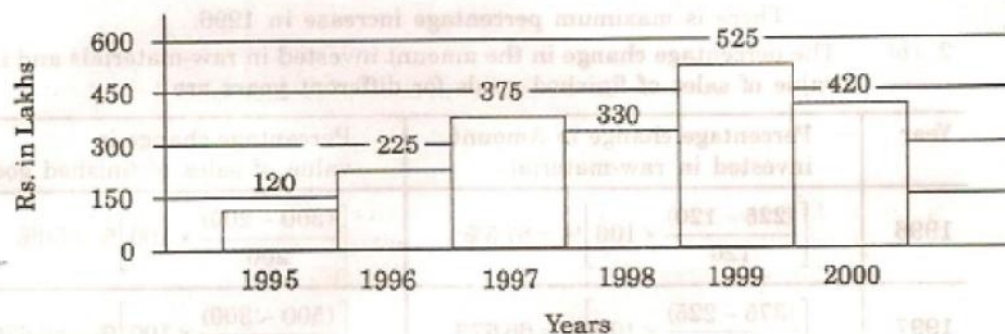
The country had reserves above 3480 million US \$ during the years 1992-93, 1996-97 and 1997-98 i.e., for 3 years and below 3480 million US \$ during the years 1991-92, 1993-94, 1994-95, 1995-96 and 1998-99 i.e., for 5 years.

Hence, required ratio = 3 : 5.

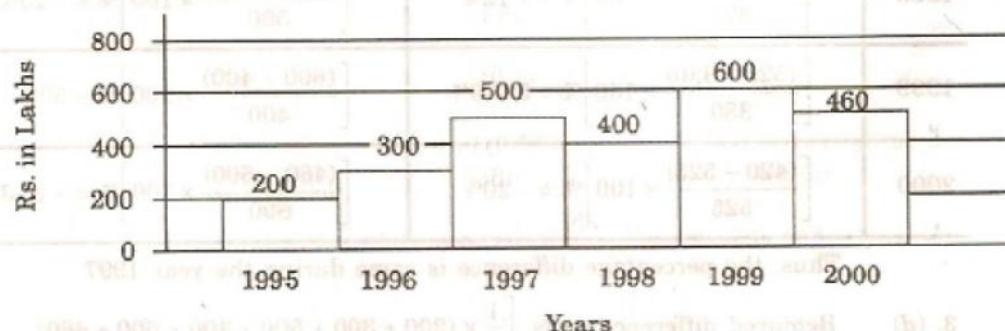
Ex. 4. Out of the two bar graphs provided below, one shows the amounts (in Lakh Rs.) invested by a Company in purchasing raw materials over the years and the other shows the values (in Lakh Rs.) of finished goods sold by the Company over the years. Study the two bar graphs and answer the questions based on them.

Amount Invested in Raw Materials and the Value of Sales of Finished Goods for a Company over the Years

Amount Invested in Raw Materials (Rs. in Lakhs)



Value of Sales of Finished Goods (Rs. in Lakhs)



- In which year, there has been a maximum percentage increase in the amount invested in Raw Materials as compared to the previous year ?
(a) 1996 (b) 1997 (c) 1998 (d) 1999 (e) 2000
- In which year, the percentage change (compared to the previous year) in the investment on Raw Materials is the same as that in the value of sales of finished goods ?
(a) 1996 (b) 1997 (c) 1998 (d) 1999 (e) 2000
- What was the difference between the average amount invested in Raw Materials during the given period and the average value of sales of finished goods during this period ?
(a) Rs. 62.5 lakhs (b) Rs. 68.5 lakhs (c) Rs. 71.5 lakhs
(d) Rs. 77.5 lakhs (e) Rs. 83.5 lakhs

4. The value of sales of finished goods in 1999 was approximately what percent of the average amount invested in Raw Materials in the years 1997, 1998 and 1999 ?
 (a) 33% (b) 37% (c) 45% (d) 49% (e) 53%
5. The maximum difference between the amount invested in Raw Materials and the value of sales of finished goods was during the year :
 (a) 1995 (b) 1996 (c) 1997 (d) 1998 (e) 1999

Sol. 1. (a) : The percentage increase in the amount invested in raw-materials as compared to the previous year, for different years are :

$$\text{For 1996} = \left[\frac{(225 - 120)}{120} \times 100 \right] \% = 87.5\%$$

$$\text{For 1997} = \left[\frac{(375 - 225)}{225} \times 100 \right] \% = 66.67\%$$

For 1998 there is a decrease.

$$\text{For 1999} = \left[\frac{(525 - 330)}{330} \times 100 \right] \% = 59.09\%$$

For 2000 there is a decrease.

\therefore There is maximum percentage increase in 1996.

2. (b) : The percentage change in the amount invested in raw-materials and in the value of sales of finished goods for different years are :

Year	Percentage change in Amount invested in raw-material	Percentage change in value of sales of finished goods
1996	$\left[\frac{(225 - 120)}{120} \times 100 \right] \% = 87.5\%$	$\left[\frac{(300 - 200)}{200} \times 100 \right] \% = 50\%$
1997	$\left[\frac{(375 - 225)}{225} \times 100 \right] \% = 66.67\%$	$\left[\frac{(500 - 300)}{300} \times 100 \right] \% = 66.67\%$
1998	$\left[\frac{(330 - 375)}{375} \times 100 \right] \% = -12\%$	$\left[\frac{(400 - 500)}{500} \times 100 \right] \% = -20\%$
1999	$\left[\frac{(525 - 330)}{330} \times 100 \right] \% = 59.09\%$	$\left[\frac{(600 - 400)}{400} \times 100 \right] \% = 50\%$
2000	$\left[\frac{(420 - 525)}{525} \times 100 \right] \% = -20\%$	$\left[\frac{(460 - 600)}{600} \times 100 \right] \% = -23.33\%$

Thus, the percentage difference is same during the year 1997.

$$\begin{aligned} \text{3. (d) : Required difference} &= \text{Rs.} \left[\frac{1}{6} \times (200 + 300 + 500 + 400 + 600 + 460) \right. \\ &\quad \left. - \frac{1}{6} \times (120 + 225 + 375 + 330 + 525 + 420) \right] \text{ lakhs} \\ &= \text{Rs.} \left[\left(\frac{2460}{6} \right) - \left(\frac{1995}{6} \right) \right] \text{ lakhs} = \text{Rs.} (410 - 332.5) \text{ lakhs} = \text{Rs.} 77.5 \text{ lakhs.} \end{aligned}$$

$$\text{4. (d) : Required percentage} = \left[\frac{600}{(375 + 330 + 525)} \times 100 \right] \% = 48.78\% \approx 49\%.$$

5. (c) : The differences between the amount invested in raw material and the value of sales of finished goods for various years are :

For 1995 = Rs. (200 - 120) lakhs = Rs. 80 lakhs.

For 1996 = Rs. (300 - 225) lakhs = Rs. 75 lakhs.

For 1997 = Rs. (500 - 375) lakhs = Rs. 125 lakhs.

For 1998 = Rs. (400 - 330) lakhs = Rs. 70 lakhs.

For 1999 = Rs. (600 - 525) lakhs = Rs. 75 lakhs.

For 2000 = Rs. (460 - 420) lakhs = Rs. 40 lakhs.

Clearly, maximum difference was during 1997.

38. PIE-CHARTS

IMPORTANT FACTS AND FORMULAE

The **pie-chart** or a **pie-graph** is a method of representing a given numerical data in the form of sectors of a circle.

The sectors of the circle are constructed in such a way that the area of each sector is proportional to the corresponding value of the component of the data.

From geometry, we know that the area of the sector of a circle is proportional to the central angle.

So, the central angle of each sector must be proportional to the corresponding value of the component.

Since the sum of all the central angles is 360° , we have

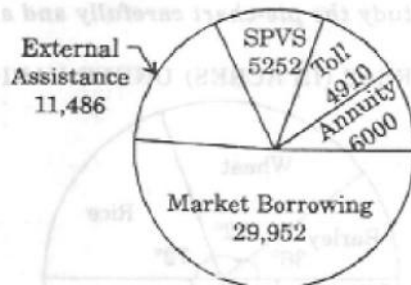
$$\text{Central angle of the component} = \left(\frac{\text{Value of the component}}{\text{Total value}} \times 360 \right)^\circ$$

SOLVED EXAMPLES

The procedure of solving problems based on pie-charts will be clear from the following solved examples.

Example 1. The following pie-chart shows the sources of funds to be collected by the National Highways Authority of India (NHAI) for its Phase II projects. Study the pie-chart and answer the questions that follow.

**SOURCES OF FUNDS TO BE ARRANGED BY NHAI
FOR PHASE II PROJECTS (IN CRORES RS.)**



Total funds to be arranged for Projects (Phase II) = Rs. 57,600 crores.

- Near about 20% of the funds are to be arranged through :
 (a) SPVS (b) External Assistance
 (c) Annuity (d) Market Borrowing
- The central angle corresponding to Market Borrowing is :
 (a) 52° (b) 137.8° (c) 187.2° (d) 192.4°
- The approximate ratio of the funds to be arranged through Toll and that through Market Borrowing is :
 (a) 2 : 9 (b) 1 : 6 (c) 3 : 11 (d) 2 : 5

4. If NHAI could receive a total of Rs. 9695 crores as External Assistance, by what percent (approximately) should it increase the Market Borrowings to arrange for the shortage of funds ?

(a) 4.5% (b) 7.5% (c) 6% (d) 8%

5. If the toll is to be collected through an outsourced agency by allowing a maximum 10% commission, how much amount should be permitted to be collected by the outsourced agency, so that the project is supported with Rs. 4910 crores ?

(a) Rs. 6213 crores (b) Rs. 5827 crores (c) Rs. 5401 crores (d) Rs. 5216 crores

SOLUTION

1. (b) : 20% of the total funds to be arranged = Rs. (20% of 57600) crores
= Rs. 11520 crores = Rs. 11486 crores.

Rs. 11486 crores is the amount of funds to be arranged through External Assistance.

2. (c) : Central angle corresponding to Market Borrowing = $\left(\frac{29952}{57600} \times 360^\circ\right) = 187.2^\circ$.

3. (b) : Required ratio = $\frac{4910}{29952} = \frac{1}{6.1} \approx \frac{1}{6}$.

4. (c) : Shortage of funds arranged through External Assistance

= Rs. (11486 - 9695) crores = Rs. 1791 crores.

∴ Increase required in Market Borrowings = Rs. 1791 crores.

Percentage increase required = $\left(\frac{1791}{29952} \times 100\right)\% = 5.98\% \approx 6\%$.

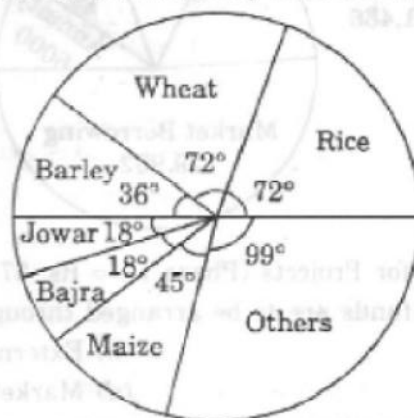
5. (c) : Amount permitted = (Funds required from Toll for projects of Phase II)
+ (10% of these funds)

= Rs. 4910 crores + Rs. (10% of 4910) crores

= Rs. (4910 + 491) crores = Rs. 5401 crores

Example 2. The pie-chart provided below gives the distribution of land (in a village) under various food crops. Study the pie-chart carefully and answer the questions that follow.

DISTRIBUTION OF AREAS (IN ACRES) UNDER VARIOUS FOOD CROPS



1. Which combination of three crops contribute to 50% of the total area under the food crops ?

(a) Wheat, Barley and Jowar (b) Rice, Wheat and Jowar
(c) Rice, Wheat and Barley (d) Bajra, Maize and Rice

2. If the total area under jowar was 1.5 million acres, then what was the area (in million acres) under rice ?

(a) 6 (b) 7.5 (c) 9 (d) 4.5

3. If the production of wheat is 6 times that of barley, then what is the ratio between the yield per acre of wheat and barley ?
 (a) 3 : 2 (b) 3 : 1 (c) 12 : 1 (d) 2 : 3
4. If the yield per acre of rice was 50% more than that of barley, then the production of barley is what percent of that of rice ?
 (a) 30% (b) $33\frac{1}{3}\%$ (c) 35% (d) 36%
5. If the total area goes up by 5%, and the area under wheat production goes up by 12%, then what will be the angle for wheat in the new pie-chart ?
 (a) 62.4° (b) 76.8° (c) 80.6° (d) 84.2°

SOLUTION

1. (c) : The total of the central angles corresponding to the three crops which cover 50% of the total area, should be 180° . Now, the total of the central angles for the given combinations are :

- (i) Wheat, Barley and Jowar = $(72^\circ + 36^\circ + 18^\circ) = 126^\circ$
 (ii) Rice, Wheat and Jowar = $(72^\circ + 72^\circ + 18^\circ) = 162^\circ$
 (iii) Rice, Wheat and Barley = $(72^\circ + 72^\circ + 36^\circ) = 180^\circ$
 (iv) Bajra, Maize and Rice = $(18^\circ + 45^\circ + 72^\circ) = 135^\circ$

Clearly, (iii) is the required combination.

2. (a) : The area under any of the food crops is proportional to the central angle corresponding to that crop.

Let, the area under rice production be x million acres.

$$\text{Then, } 18 : 72 = 1.5 : x \Rightarrow x = \left(\frac{72 \times 1.5}{18} \right) = 6.$$

Thus, the area under rice production = 6 million acres.

3. (b) : Let the total production of barley be T tonnes and let Z acres of land be put under barley production.

Then, the total production of wheat = $(6T)$ tonnes.

Also, area under wheat production = $(2Z)$ acres.

$$\left[\begin{aligned} \therefore \frac{\text{Area under Wheat production}}{\text{Area under Barley production}} &= \frac{72^\circ}{36^\circ} = 2 \\ \text{and therefore, Area under wheat} &= 2 \times \text{Area under barley} = (2Z) \text{ acres} \end{aligned} \right]$$

$$\text{Now, yield per acre for wheat} = \left(\frac{6T}{2Z} \right) \text{ tonnes/acre} = \left(\frac{3T}{Z} \right) \text{ tonnes/acre}$$

$$\text{and yield per acre for barley} = \left(\frac{T}{Z} \right) \text{ tonnes/acre.}$$

$$\therefore \text{Required Ratio} = \left(\frac{3T/Z}{T/Z} \right) = 3 : 1.$$

4. (b) : Let Z acres of land be put under barley production.

$$\text{Then, } \frac{\text{Area under rice production}}{\text{Area under barley production}} = \frac{72^\circ}{36^\circ} = 2.$$

\therefore Area under Rice production = $2 \times$ area under barley production = $(2Z)$ acres.

Now, if p tonnes be the yield per acre of barley then, yield per acre of rice

$$= (p + 50\% \text{ of } p) \text{ tonnes} = \left(\frac{3}{2} p \right) \text{ tonnes.}$$

∴ Total production of rice = (yield per acre) × (area under production)
 $= \left(\frac{3}{2}p\right) \times 2Z = (3pZ)$ tonnes.

And, Total production of barley = (pZ) tonnes.

∴ Percentage production of barley to that of rice = $\left(\frac{pZ}{3pZ} \times 100\right)\% = 33\frac{1}{3}\%$.

5. (b) : Initially, let t acres be the total area under consideration.

Then, area under wheat production initially was = $\left(\frac{72}{360} \times t\right)$ acres = $\left(\frac{t}{5}\right)$ acres.

Now, if the total area under consideration be increased by 5%, then the new

value of the total area = $\left(\frac{105}{100}t\right)$ acres.

Also, if the area under wheat production be increased by 12%, then the new

value of the area under wheat = $\left[\frac{t}{5} + \left(12\% \text{ of } \frac{t}{5}\right)\right]$ acres = $\left(\frac{112t}{500}\right)$ acres.

∴ Central angle corresponding to wheat in the new pie-chart

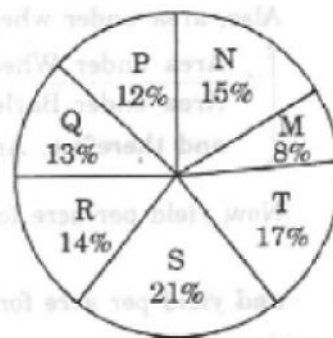
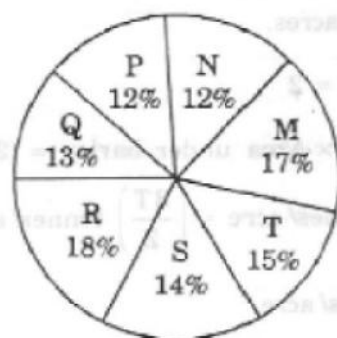
$$= \left[\frac{\text{Area under wheat (new)}}{\text{Total area (new)}} \times 360 \right]^\circ = \left[\frac{\left(\frac{112t}{500}\right)}{\left(\frac{105t}{100}\right)} \times 360 \right]^\circ = 76.8^\circ.$$

Example 3. The following pie-charts show the distribution of students of graduate and post-graduate levels in seven different institutes — M, N, P, Q, R, S and T in a town. (Bank P.O. 2003)

DISTRIBUTION OF STUDENTS AT GRADUATE AND POST-GRADUATE LEVELS IN SEVEN INSTITUTES — M, N, P, Q, R, S AND T

Total Number of Students of
Graduate Level = 27300

Total Number of Students of
Post-Graduate Level = 24700



- How many students of institutes M and S are studying at graduate level ?
 (a) 7516 (b) 8463 (c) 9127 (d) 9404
- Total number of students studying at post-graduate level from institutes N and P is :
 (a) 5601 (b) 5944 (c) 6669 (d) 7004
- What is the total number of graduate and post-graduate level students in institute R ?
 (a) 8320 (b) 7916 (c) 9116 (d) 8372
- What is the ratio between the number of students studying at post-graduate and graduate levels respectively from institute S ?
 (a) 14 : 19 (b) 19 : 21 (c) 17 : 21 (d) 19 : 14

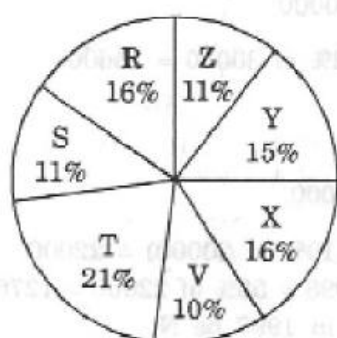
5. What is the ratio between the number of students studying at post-graduate level from institute S and the number of students studying at graduate level from institute Q ?
 (a) 13 : 19 (b) 21 : 13 (c) 13 : 8 (d) 19 : 13

SOLUTION

- (b) : Students of institute M at graduate level = 17% of 27300 = 4641.
 Students of institute S at graduate level = 14% of 27300 = 3822.
 \therefore Total number of students at graduate level in institutes M and S
 $= 4641 + 3822 = 8463$.
- (c) : Required number = (15% of 24700) + (12% of 24700) = 3705 + 2964 = 6669.
- (d) : Required number = (18% of 27300) + (14% of 24700) = 4914 + 3458 = 8372.
- (d) : Required ratio = $\frac{(21\% \text{ of } 24700)}{(14\% \text{ of } 27300)} = \frac{21 \times 24700}{14 \times 27300} = \frac{19}{14}$.
- (d) : Required ratio = $\frac{(21\% \text{ of } 24700)}{(13\% \text{ of } 27300)} = \frac{21 \times 24700}{13 \times 27300} = \frac{19}{13}$.

Example 4. Study the following pie-chart and the table and answer the questions asked on them. (S.B.I.P.O. 1999)

PROPORTION OF POPULATION OF SEVEN VILLAGES IN 1997



Village	% Population Below Poverty Line
X	38
Y	52
Z	42
R	51
S	49
T	46
V	58

- Find the population of village S if the population of village X below poverty line in 1997 is 12160.
 (a) 18500 (b) 20500 (c) 22000 (d) 26000
- The ratio of population of village T below poverty line to that of village Z below poverty line in 1997 is :
 (a) 11 : 23 (b) 13 : 11 (c) 23 : 11 (d) 11 : 13
- If the population of village R in 1997 is 32000, then what will be the population of village Y below poverty line in that year ?
 (a) 14100 (b) 15600 (c) 16500 (d) 17000
- If in 1998, the population of villages Y and V increase by 10% each and the percentage of population below poverty line remains unchanged for all the villages, then find the population of village V below poverty line in 1998, given that the population of village Y in 1997 was 30000.
 (a) 11250 (b) 12760 (c) 13140 (d) 13780

5. If in 1999, the population of village R increases by 10% while that of village Z reduces by 5% compared to that in 1997 and the percentage of population below poverty line remains unchanged for all the villages, then find the approximate ratio of population of village R below poverty line to the ratio of population of village Z below poverty line for the year 1999.

(a) 2 : 1

(b) 3 : 2

(c) 4 : 3

(d) 5 : 4

SOLUTION

1. (c) : Let the population of village X be x .

$$\text{Then, } 38\% \text{ of } x = 12160 \Rightarrow x = \frac{12160 \times 100}{38} = 32000.$$

Now, if s be the population of village S, then

$$16 : 11 = 32000 : s \Rightarrow s = \frac{11 \times 32000}{16} = 22000.$$

2. (c) : Let N be the total population of all the seven villages.

Then, population of village T below poverty line = 46% of (21% of N)

and population of village Z below poverty line = 42% of (11% of N)

$$\therefore \text{Required ratio} = \frac{46\% \text{ of } (21\% \text{ of } N)}{42\% \text{ of } (11\% \text{ of } N)} = \frac{46 \times 21}{42 \times 11} = \frac{23}{11}.$$

3. (b) : Population of village R = 32000 (given).

Let the population of village Y be y .

$$\text{Then, } 16 : 15 = 32000 : y \Rightarrow y = \frac{15 \times 32000}{16} = 30000$$

\therefore Population of village Y below poverty line = 52% of 30000 = 15600.

4. (b) : Population of village Y in 1997 = 30000 (given).

Let the population of village V in 1997 be v .

$$\text{Then, } 15 : 10 = 30000 : v \Rightarrow v = \frac{30000 \times 10}{15} = 20000.$$

Now, population of village V in 1998 = 20000 + (10% of 20000) = 22000.

\therefore Population of village V below poverty line in 1998 = 58% of 22000 = 12760.

5. (a) : Let the total population of all the seven villages in 1997 be N .

$$\text{Then, population of village R in 1997} = 16\% \text{ of } N = \frac{16}{100} N$$

$$\text{and population of village Z in 1997} = 11\% \text{ of } N = \frac{11}{100} N.$$

$$\therefore \text{Population of village R in 1999} = \left\{ \frac{16}{100} N + \left(10\% \text{ of } \frac{16}{100} N \right) \right\} = \frac{1760}{10000} N$$

$$\text{and population of village Z in 1999} = \left\{ \frac{11}{100} N - \left(5\% \text{ of } \frac{11}{100} N \right) \right\} = \frac{1045}{10000} N$$

$$\text{Now, population of village R below poverty line for 1999} = 51\% \text{ of } \left(\frac{1760}{10000} N \right)$$

$$\text{and population of village Z below poverty line for 1999} = 42\% \text{ of } \left(\frac{1045}{10000} N \right).$$

$$\therefore \text{Required ratio} = \frac{51\% \text{ of } \left(\frac{1760}{10000} N \right)}{42\% \text{ of } \left(\frac{1045}{10000} N \right)} = \frac{51 \times 1760}{42 \times 1045} = \frac{2}{1}.$$

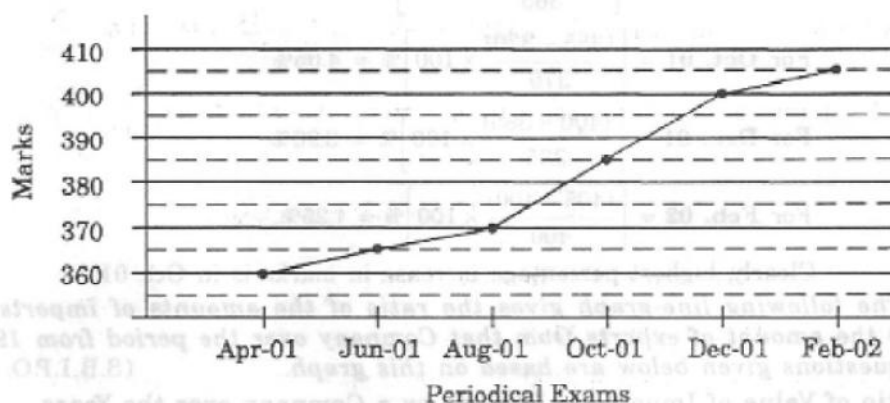
39. LINE-GRAPHS

This section comprises of questions in which the data collected in a particular discipline are represented by specific points joined together by straight lines. The points are plotted on a two-dimensional plane taking one parameter on the horizontal axis and the other on the vertical axis. The candidate is required to analyse the given information and thereafter answer the given questions on the basis of the analysis of data.

SOLVED EXAMPLES

Ex. 1. In a school the periodical examinations are held every second month. In a session during Apr. 2001 – Mar. 2002, a student of Class IX appeared for each of the periodical exams. The aggregate marks obtained by him in each periodical exam are represented in the line-graph given below. Study the graph and answer the questions based on it. (S.B.I.P.O. 2003)

MARKS OBTAINED BY A STUDENT IN SIX PERIODICAL EXAMS HELD IN EVERY TWO MONTHS DURING THE YEAR IN THE SESSION 2001-02
Maximum Total Marks in each Periodical Exam = 500



- The total number of marks obtained in Feb. 02 is what percent of the total marks obtained in Apr. 01 ?
 (a) 110% (b) 112.5% (c) 115% (d) 116.5% (e) 117.5%
 - What are the average marks obtained by the student in all the periodical exams during the session ?
 (a) 373 (b) 379 (c) 381 (d) 385 (e) 389
 - What is the percentage of marks obtained by the student in the periodical exams of Aug. 01 and Oct. 01 taken together ?
 (a) 73.25% (b) 75.5% (c) 77% (d) 78.75% (e) 79.5%
 - In which periodical exams there is a fall in percentage of marks as compared to the previous periodical exams ?
 (a) None (b) Jun. 01 (c) Oct. 01 (d) Feb. 02 (e) None of these
 - In which periodical exams did the student obtain the highest percentage increase in marks over the previous periodical exams ?
 (a) Jun. 01 (b) Aug. 01 (c) Oct. 01 (d) Dec. 01 (e) Feb. 02
- Sol.** Here it is clear from the graph that the student obtained 360, 365, 370, 385, 400 and 405 marks in periodical exams held in Apr. 01, Jun. 01, Aug. 01, Oct. 01, Dec. 01 and Feb. 02 respectively.

1. (b) : Required percentage = $\left(\frac{405}{360} \times 100\right)\% = 112.5\%$.

2. (c) : Average marks obtained in all the periodical exams

$$= \frac{1}{6} \times [360 + 365 + 370 + 385 + 400 + 405] = 380.83 \approx 381.$$

3. (b) : Required percentage = $\left[\frac{(370 + 385)}{(500 + 500)} \times 100\right]\% = \left(\frac{755}{1000} \times 100\right)\% = 75.5\%$.

4. (a) : As is clear from the graph, the total marks obtained in periodical exams, go on increasing. Since, the maximum marks for all the periodical exams are same, it implies that the percentage of marks also goes on increasing. Thus, in none of the periodical exams, there is a fall in percentage of marks compared to the previous exam.

5. (c) : Percentage increase in marks in various periodical exams compared to the previous exams are :

For Jun. 01 = $\left[\frac{(365 - 360)}{360} \times 100\right]\% = 1.39\%$

For Aug. 01 = $\left[\frac{(370 - 365)}{365} \times 100\right]\% = 1.37\%$

For Oct. 01 = $\left[\frac{(385 - 370)}{370} \times 100\right]\% = 4.05\%$

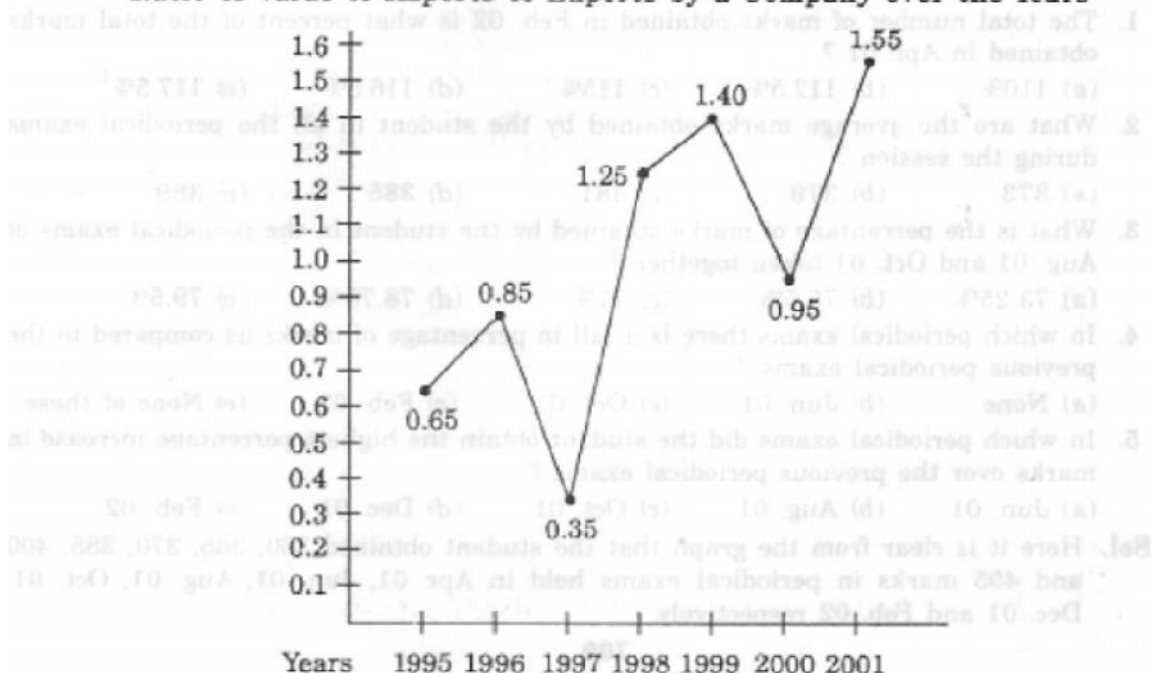
For Dec. 01 = $\left[\frac{(400 - 385)}{385} \times 100\right]\% = 3.90\%$

For Feb. 02 = $\left[\frac{(405 - 400)}{400} \times 100\right]\% = 1.25\%$.

Clearly, highest percentage increase in marks is in Oct. 01.

Ex. 2. The following line-graph gives the ratio of the amounts of imports by a Company to the amount of exports from that Company over the period from 1995 to 2001. The questions given below are based on this graph. (S.B.I.P.O. 2001)

Ratio of Value of Imports to Exports by a Company over the Years



1. In how many of the given years were the exports more than the imports ?
(a) 1 (b) 2 (c) 3 (d) 4 (e) None of these
2. The imports were minimum proportionate to the exports of the Company in the year :
(a) 1995 (b) 1996 (c) 1997 (d) 2000 (e) 2001
3. If the imports of the Company in 1996 was Rs. 272 crores, the exports from the Company in 1996 was :
(a) Rs. 370 crores (b) Rs. 320 crores (c) Rs. 280 crores
(d) Rs. 275 crores (e) Rs. 264 crores
4. What was the percentage increase in imports from 1997 to 1998 ?
(a) 72 (b) 56 (c) 28 (d) None of these (e) Data inadequate
5. If the imports in 1998 was Rs. 250 crores and the total exports in the years 1998 and 1999 together was Rs. 500 crores, then the imports in 1999 was :
(a) Rs. 250 crores (b) Rs. 300 crores (c) Rs. 357 crores
(d) Rs. 420 crores (e) None of these

Sol. 1. (d) : The exports are more than the imports implies that the ratio of value of imports to exports is less than 1.

Now, this ratio is less than 1 in the years 1995, 1996, 1997 and 2000.

Thus, there are four such years.

2. (c) : The imports are minimum proportionate to the exports implies that the ratio of the value of imports to exports has the minimum value.

Now, this ratio has a minimum value of 0.35 in 1997, i.e., the imports are minimum proportionate to the exports in 1997.

3. (b) : Ratio of imports to exports in the year 1996 = 0.85.

Let the exports in 1996 = Rs. x crores.

$$\text{Then, } \frac{272}{x} = 0.85 \Rightarrow x = \frac{272}{0.85} = 320.$$

\therefore Exports in 1996 = Rs. 320 crores.

4. (e) : The graph gives only the ratio of imports to exports for different years. To find the percentage increase in imports from 1997 to 1998, we require more details such as the value of imports or exports during these years. Hence, the data is inadequate to answer this question.

5. (d) : The ratio of imports to exports for the years 1998 and 1999 are 1.25 and 1.40 respectively.

Let the exports in the year 1998 = Rs. x crores.

Then, the exports in the year 1999 = Rs. $(500 - x)$ crores.

$$\therefore 1.25 = \frac{250}{x} \Rightarrow x = \frac{250}{1.25} = 200 \quad [\text{Using ratio for 1998}]$$

Thus, the exports in the year 1999 = Rs. $(500 - 200)$ crores = Rs. 300 crores.

Let the imports in the year 1999 = Rs. y crores.

$$\text{Then, } 1.40 = \frac{y}{300} \Rightarrow y = (300 \times 1.40) = 420.$$

\therefore Imports in the year 1999 = Rs. 420 crores.